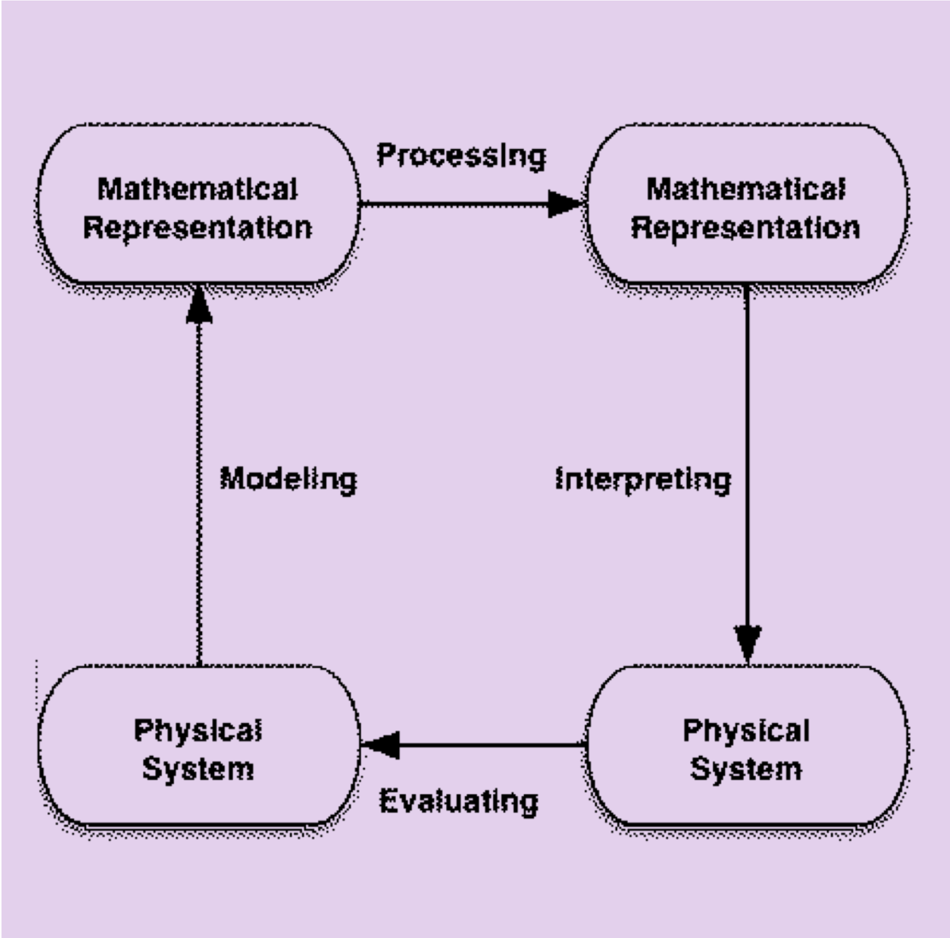


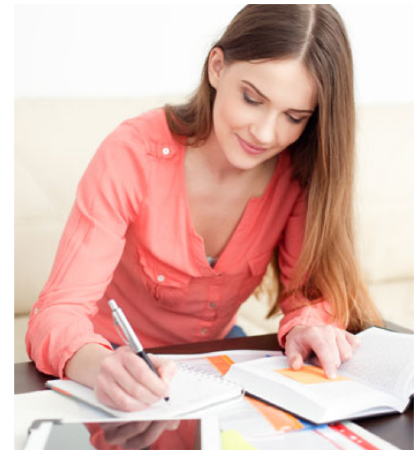
Using math in introductory physics:

They're measurements, not numbers!

Edward Redish, Mark Eichenlaub, and Deborah Hemingway
Department of Physics, U of MD

Teaching physics to bio majors and pre-meds

- Many of my students are successful in calculus classes



but freak out when called on to do algebra in physics.

Why?

Math in most math classes

$\neq$

How math is used in physics

In physics we use math to think with.

We blend our physical knowledge with math, changing the way we think about the math.*

* Redish and Kuo, *Sci. and Educ.*, 24:5-6, 561-590 (2015)

Failing to blend the math and the physics has many implications.

- Students focus on numerical calculations, and don't "think with math" as they need to do in physics.
- They treat measurements as if they were numbers, undermining their ability to build a connection between physics and math.
- They have a strong resistance to working with symbols and using dimensions and units.

A key point is that the symbols we work with do **not** stand for numbers but for measurements.

Significant and insignificant figures

A math task:

What is 1.843 x 3.686?

1.843 x 3.686 = 6.793298

A physics task:

What is the area of a room

1.843 m x 3.686 m?

Could be

1.8434 x 3.6864 = 6.79550976

or

1.8426 x 3.6857 = 6.79127082

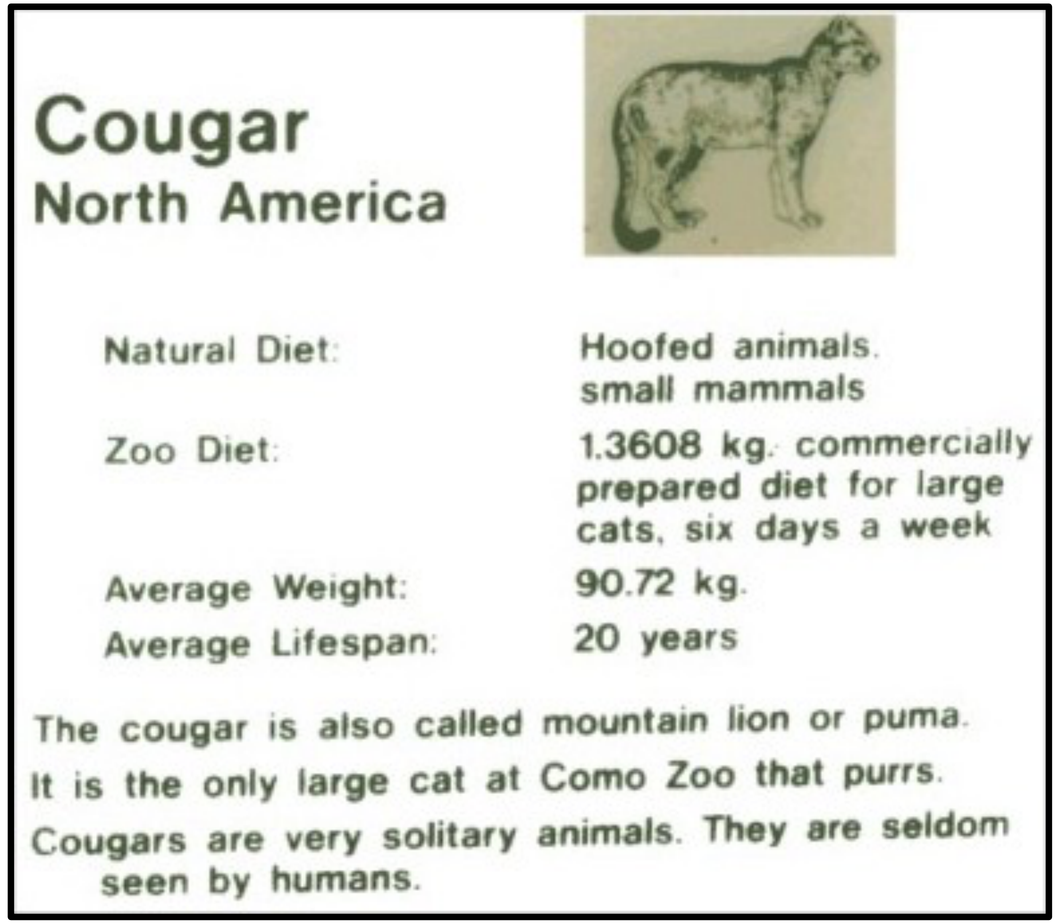
Therefore: report answer as 6.79 m^2

These digits are INSIGNIFICANT FIGURES.

Feeding the cougar

I saw this sign when taking my grandchildren to the Como Park Zoo in Minneapolis.

Why is the lifespan of the cougar listed rounded to 10 years but its daily diet is specified to a tenth of a gram?



Dimensional analysis (DA)

- The traditional notation for DA is very confusing. It looks like algebra but it isn't.

$$[\Delta x] = L$$

$$[\Delta t] = T$$

$$[m] = M$$

- But then

$$[\Delta x_1 + \Delta x_2] = [\Delta x_1] + [\Delta x_2] = L + L = L$$

- Traditional DA brackets are really asking the question: What measuring tools were used to get this number?

Introduce DA with a different kind of bracket using icons – not symbols.

$$[\![\Delta x]\!] =$$

$$[\![\Delta t]\!] =$$

$$[\![m]\!] =$$

$$[\![T]\!] =$$

$$[\![Q]\!] =$$

The key to making it work: Keep at it!

- I introduce the idea "values in science are measurements not numbers" early and push dimensional analysis throughout the class.
- In the first semester, in addition to it appearing regularly in homework there were DA problems on
  - 5 of the weekly quizzes
  - 3 questions on the two midterm exams
  - 1 question on the final
- Many problems involved equations that were extensions of what they had seen or were totally new to them.

Generic volume formula question using DA and special cases

Some quiz questions during the first semester

2. (3 points) A trapezoidal prism has two of its sides as trapezoids and the rest are rectangles. It looks something like the figure at the right. Which of the following expressions could potentially be the correct formula for the volume of this prism?

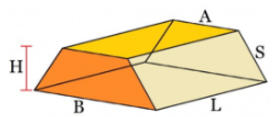
A. $\frac{1}{3}ABSLH$

B. $\frac{1}{4}L(A+B)\sqrt{4S^2+2AB-A^2-B^2}$

C. $(A+B+2S)L+(A+B)H$

D. $L(A+B)\sqrt{4S^2+2AB-A^2-B^2}$

E. None of these could possibly be correct.



Generic volume formula question using DA and special cases

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A. $\frac{1}{3}$

B. $\frac{1}{4}$

C. (

D. L

E. N

DA of "the jerk"
(time derivative of acceleration

1. (3 points) In our description of motion, if we have a position as a function of time, $x(t)$, we defined its derivative as the velocity.

$$v(t) = \frac{dx(t)}{dt} .$$

We can also create higher derivatives. The derivative of the velocity is the acceleration, and the derivative of the acceleration called the *jerk*, $j(t)$. (Really!)

$$a(t) = \frac{dv(t)}{dt} \qquad j(t) = \frac{da(t)}{dt}$$

What is the dimensionality of j?

a. $[j] = \mathrm{LT}$
b. $[j] = \mathrm{L}$
c. $[j] = \mathrm{L/T}$
d. $[j] = \mathrm{L/T^2}$
e. $[j] = \mathrm{L/T^3}$
f. You can't tell from the information given
g. Something else. (What? Put it in the box.)

Some quiz questions during the first semester

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We can also c… …celeration, and the derivative of the accelera…

What is the di…

DA of concentration gradient (for Fick's law of diffusion)

1. (2 points) In this class, we often introduce variables that are the derivatives of other variables. The velocity is an obvious one. Another is the *concentration gradient* – how the concentration of a particular chemical changes with position: dn/dx. In this case, n has the dimensions of number/volume, or $[n] = 1/L^3$. The position, x, of course has dimensions of length, $[x] = L$. What are the dimensions of the derivative $[dn/dx]$?

a. $1/L^2$
b. $1/L^3$
c. $1/L^4$
d. Something else. (What? Put it in the box with your answer.)
e. The answer cannot be determined without more information

Some quiz questions during the first semester

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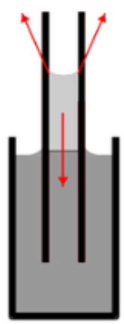
DA of capillary action equation (not covered in class)

3. (3 points) The liquid in a capillary tube crawls up the tube because the liquid sticks to the glass of the tube. But the height it climbs up to is determined by the surface tension of the water. (The figure shows the raised bit of water in lighter gray for identification purposes.) Essentially, the very highest bit of water pulls up on the rest of the water by surface tension (upward arrows in the figure). The upward pull of the surface tension balances the weight of the water pulled up. We therefore expect that the height that water rises in a capillary depends on: the surface tension of the water, γ (gamma), the density of water, ρ (rho), the gravitational field, g, and the radius of the capillary tube, r. Which of these expressions could be a correct expression for the height water rises in a capillary (ignoring dimensionless factors like 2, π, or cos θ).



A. $h = \frac{\gamma}{\rho g r^2}$

B. $h = \gamma \rho g r$

C. $h = \frac{\gamma}{\rho g r}$

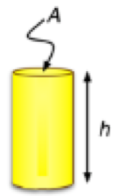
D. $h = \frac{\gamma r}{\rho g}$

E. None of these

Exam question – multiple DA part (20%)

Some exam questions during the first semester

5. (20 points) Throughout this course, we try to analyze the measured properties of specific objects in terms of their geometric shape and a parameter that just depends on the materials that they are made of. In this problem we'll generate equations connecting these measurements. For this problem, we are just interested in the dimensioned variables, so if you think there are dimensionless numerical constants (like 2, π, or 10^{23}, don't bother to put them in.)



1. (5 pts) One example that you are already familiar with is density. Suppose a particular cylinder (not necessarily circular) has an area A and a height h. The property of the material that doesn't depend on its shape is the **density**, ρ (Gk. "rho" in physics). The cylinder's mass can be written as an equation. Write an equation for the mass, m, of the cylinder in terms of the cylinder's area, A, height, h, and the density of the material of which it's made, ρ. Show how you got your equation using dimensional analysis.

$m =$

2. (5 pts) If our cylinder is a pipe with fluid flowing in it at a(n approximately) constant rate, because the internal viscosity of the fluid tends to slow the fluid down, there has to be an external force that balances it (by N2). This force is produced by a pressure drop along the direction of fluid flow. If we write J for the fluid flowing in the pipe (where J is the volume of fluid moving into or out of the cylinder per second), then the flow in the pipe is proportional to the difference between the forces at the two ends. We'll call the proportionality constant "R" for **resistance**. The equation then has the form

$$\Delta F = RJ$$

where $\Delta F = F_2 - F_1$. Find the dimensionality of R in terms of the dimensions M, L, and T.

$[R] =$

3. (10 pts) Another property of the cylinder is how it responds to being pressed straight down on its top. Most objects will compress at least a little. The amount of compression, Δh, depends on the force, F_0, that is pressing down on A. (F_0 is a perpendicular normal force.) The general property of the material of which the cylinder is made that controls this is called **Young's modulus**, Y, and it has the dimensions of force/area. Write an equation for the amount of compression, Δh, of the cylinder in terms of the Young's modulus, the force, F_0, the cylinder's area, A, and height, h. Show how you got your equation using dimensional analysis.

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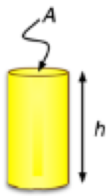

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Exam question – essay on DA

4. (10 points) You have used dimensions several different ways in this class. For example, you have derived new formulas by combining variables in the only dimensionally-consistent way. Please list two or more different uses for dimensions in physics (you can include the one already mentioned here). Then choose one of those uses and explain why it is useful. Specifically, does it tell you everything about how to solve a physics problem, or just part? Give a specific example to validate your point. *Note: This is an essay question. Your answer will be judged not solely on its correctness, but for its depth, coherence, and clarity.*

2. (5 pts) If our ... internal viscosi... it (by N2). This ... the fluid flowi... second), then t... We'll call the proportionality constant "R" for **resistance**. The equation then has the form

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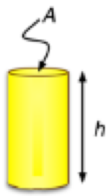

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Final Exam Question – DA of a system we hadn't discussed

2. (10 points) So far, we have studied two equations with "gradient driven flow"; the H-P Equation and Fick's law. In these equations, a change in some scalar variable in space (pressure, concentration), results in a flow of something (fluid, dissolved chemicals). Another such equation is Fourier's heat transfer law:

$$\Phi = -z\frac{\Delta(k_B T)}{\Delta x}$$

(where k_B is Boltzmann's constant). This says that if there is a change in temperature in space, there will be a flow, Φ, of thermal energy per unit area per second that is proportional to the temperature gradient. (We've put in a factor of Boltzmann's constant to simplify the calculation for you.)
What are the dimensions of the proportionality constant z? Give your answer using the dimensional terms mass, length, time and temperature (M, L, T and Θ).

An emphasis on DA was a key part of an all fronts effort to get bio majors and pre-meds to take symbolic reasoning more seriously. Did it work?

On the last day, on an anonymous clicker question, I asked “Do you think your skills in reading symbolic expressions (equations) and using them to understand physical situations has improved in this class?”

- 48% responded “I have improved dramatically”.
- 41% responded, “I have improved a little”.

Many students reported that DA was one of the most important things they learned during the semester and that it was very valuable in preparing for MCATs.

To see more components of our effort to improve math use for bio majors and pre-meds,
see our PERC poster

and the website with some of our materials:
https://www.compadre.org/nexusph/course/Modeling_with_mathematics