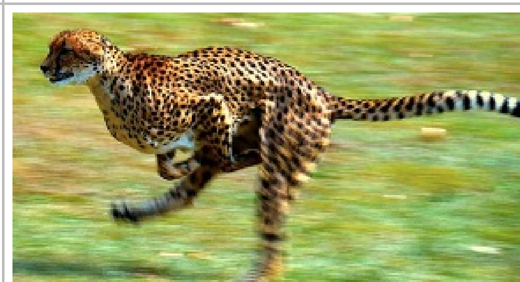
umdberg: Kinematics (open):The cat and the antelope

One of the classic examples of predator-prey is the big cat and the herbivore. Like many predators and their prey, natural selection leads them on an "arms race", with the faster variants on both sides having a survival advantage, leading to both the prey and the predator being able to run at very high speeds. The actual interaction between predator and prey is quite complex, involving not just speed, but the evolution of stamina, jumping, pouncing, and camouflage.* For this problem, let's just model one aspect of the predator prey interaction by considering a simple question. How far away is safe enough?

Fast herbivores that flee fast predators like to avoid locations where a camouflaged predator can hide. How far from a dangerous area should a prey animal stay in order to be able to escape if a predator bursts into view? This distance depends on how the animal runs. Let's consider a specific example.

A cheetah is one of the fastest animals, but it can only maintain its high speed for a short time. The Thomson's gazelle can continue to run at a steady pace for a long time, but its maximum speed is less than that of the cheetah. Here are some of the parameters of their motions.

Thomson's gazelle: • Acceleration: from 0 to 90 km/hr in about 18 seconds. • Max speed: 90 km/hr • Stamina: Can maintain its maximum speed for long periods of time (compared to the cheetah).	 Photo by David Bygott, CC permission
Cheetah: • Acceleration: from 0 to 120 km/hr in 3 seconds • Max speed: 120 km/hr but can only maintain it for about 30 seconds. • After its initial high speed sprint, it collapses due to heat exhaustion -- its temperature may rise as high as 105^o F -- and itself becomes vulnerable to larger predators.	 Photo by Jason Bechtel CC permission

A. Let's begin by considering a very simple model of predator-prey interaction. Let's assume that the predator creeps up on a resting prey to a distance, d, and catches the prey during the period when they are both accelerating. At time t = 0, the prey sees the predator and both take off, accelerating as fast as they can.

A.1 The cheetah only accelerates for about 3 seconds. If they both start accelerating at the same time, what will the gazelle's and cheetah's average velocity be at the end of that time interval?

A.2 How far will they each have traveled in that time interval? How far away does the gazelle have to start in order for the cheetah not to catch it during its acceleration phase?

B. Now let's refine the model by including the cheetah's high speed sprint as well as its acceleration. We notice that the cheetah can run faster than the gazelle, but only for 30 seconds. If they are both running straight, how far away must the gazelle be from the cheetah in order that the cheetah not catch it before the cheetah collapses from exhaustion?

C. Find a symbolic representation of the maximum distance, d, for which a general predator can catch its prey if:

- The predator can accelerate at a rate of a_1 for a time t_1 and can run at that final speed for a time T_1. After that time, it must stop running.
- The prey can accelerate at a rate of a_2 for a time t_2 and can run at that final speed for a long time -- at least for a lot longer than the predator can run.

Check your final answer by seeing if the values you got for part B are obtained by putting the appropriate parameters into your equation. Consider various limiting cases to see if your equation makes sense.

D. The equations you have generated for part C have been turned into a spreadsheet simulation that creates a graph showing the position of the cheetah and the gazelle as a function of time given values for the parameters a_1, t_1, T_1, a_2, t_2, and d. Download the file cheetahgazelle.xls and explore the dependence of the resulting paths on one of the parameters. Describe in words something interesting that you have learned about the chase from this exploration and illustrate your conclusion with printouts of a few graphs.

* R, McNeill Alexander, *Principles of Animal Locomotion* (Princeton University Press, 2003), Chapter 1.

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