

**nexus**  
NATIONAL EXPERIMENT  
in Undergraduate Science Education

**Teaching  
Physics  
to Biologists**

UNIVERSITY OF  
MARYLAND  
HHMI



NEXUS is a collaborative project to develop a new national science curriculum for biologist and pre-health care professionals.

- UMBC (Math in Bio)
- UMCP (Physics)
- Purdue University (Chemistry) and
- The University of Miami (Interdisciplinary case studies)

UMBC  
AN HONORS  
UNIVERSITY  
IN MARYLAND



**PURDUE**  
UNIVERSITY.

**UNIVERSITY OF MIAMI**

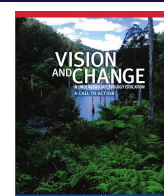
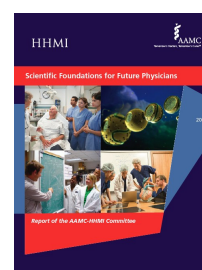
NEXUS Workshop

1/11/12



The focus of the project is the AAMC-HHMI Report: *Scientific Foundations for Future Physicians* (2009)

This report called for reform of science education for medical, pharmacy, and veterinary schools and all students who are studying the basic biological sciences.



NEXUS Workshop

1/11/12



## Goals of NEXUS: A national demonstration project

- Create prototype materials
  - an inventory of instructional modules that can be shared nationally as open source materials.
- Interdisciplinary
  - Coordinate instruction in biology, chemistry, physics, and math.
- Competency based
  - Teach generalized scientific skills in a way that supports instruction in the other disciplines.

4

NEXUS Workshop

1/11/12

## + Developers and discussants

5

- **Physicists**
  - Joe Redish
  - Wolfgang Losert
  - Eun-Suk Seo
  - Catherine Crouch (Swarthmore)
  - Jessica Watkins
  - Chandra Turpen
  - Vashti Sawtelle
  - Ben Dreyfus
  - Michael Fisher
  - Peter Shawhan
  - Arnaldo Vaz (Brazil)
- **Chemists**
  - Bonnie Dixon
  - Chris Bauer (UNH)
  - Melanie Cooper (Clemson)
- **Biologists**
  - Todd Cooke
  - Jeff Jensen
  - Karen Carleton
  - Joelle Presson
  - Kaci Thompson
  - Marco Colombini
  - Kristi Hall-Burke
  - Mike Klymkowski (Colorado)
- **Education Specialists**
  - Janet Coffey
  - Dan Levin
  - Jen Richards
  - Julia Svoboda
  - Gili Marbach-Ad

NEXUS Workshop

1/11/12

## + Can we teach physics to biologists in a way that adds value for them?

6

- What content should we teach?
- What are the barriers to constructing an effective course?
- What do we need to do to create effective inter- or trans-disciplinary instruction?

NEXUS Workshop

1/11/12

## + Explicit Goals of the Physics Course

1/11/12

NEXUS Workshop

7

## + General Goals

8

- To redesign the Physics for Biologists course so that it is appropriate for biology students – both in content and in skill development.
- To include authentic biological examples in which students see the use (and methods) of physics as helpful in helping them make sense of something important in biology.
- To have biology faculty teaching the upper division bio classes (neuroscience, cellular bio, mammalian physiology, etc.) see this course as a desirable prerequisite to their classes.

NEXUS Workshop

1/11/12

## + Non-goal

9

- The course is NOT intended to serve as a filter to exclude students from going on to medical school or in biology research.

NEXUS Workshop

1/11/12

## + More specific goals

10

- To help students learn to reason mathematically, including to blend mathematical representations and conceptual physics.
- To help students learn to understand scientific modeling.
- To help students learn to quantify their experience (estimation).
- To help students learn to imbed and see the implications of physical mechanism in biology.
- To help students learn to reason from basic principles and seek consistency (suppressing one-step recall).

NEXUS Workshop

1/11/12



## Timetable

11

- **Content negotiation (spring and summer 2011)**
  - Where do you start? Informed by physics education literature.
  - Where do you want to end up? Informed by biologists “wish list” and position papers.
  - Consensus building -> syllabus
- **Fleshing out content (summer and fall 2011)**
  - Physical principle
  - Macro scale (organismal/systems biology) context/application
  - Micro scale (cell/molecular) context/application
  - Competency (from SFFP and Vision & Change)

NEXUS Workshop

1/11/12



## Timetable

12

- **Development of instructional materials and assessment instruments (summer and fall 2011, spring 2012)**
  - Pre-class readings
  - Lectures
  - Active learning elements (e.g., clicker questions)
  - Group problem solving activities (for lecture and recitations)
  - Homework problems
  - Formative and summative assessment instruments
- **Pilot (2011-2012 academic year)**
- **Fine tuning of content, sequence, instructional materials, and implementation (2012)**
- **Depending on progress, larger pilot or full implementation during 2012-2013 academic year**

NEXUS Workshop

1/11/12



## Content decisions: Expand or include discussion of

13

- Atomic and molecular models of matter
- Energy, including chemical energy
- Fluids, including fluids in motion and solutions
- Diffusion and gradient driven flows
- More emphasis on dissipative forces (viscosity)
- Electrostatics in fluids
- Kinetic theory, implications of random motion, statistical picture of thermodynamics
  - Non-equilibrium thermo???!?

NEXUS Workshop

1/11/12



## Content decisions: Reduce or eliminate discussion of

14

- Projectile motion
- Universal gravitation
- Inclined planes, mechanical advantage
- Linear momentum
- Rotational motion
- Torque, statics, and angular momentum

NEXUS Workshop

1/11/12

## + Implication

15

- The course is configured as a *second year* class so it can serve as a pre-requisite for upper division bio classes.
- The course has unusual pre-requisites:
  - Two semesters of bio including some cellular bio, biochem, genetics, and evolution.
  - One semester of chemistry
  - Two semesters of math including one-variable calculus and basic elements of probability.
- We are currently teaching a first version of this class to ~20 students.

NEXUS Workshop

1/11/12

## + Interdisciplinary authenticity

1/11/12

NEXUS Workshop

16

## + Conjecture

17

- One reason for the difficulty of getting more physics into bio and vice versa is epistemological – having to do with the nature and the structuring of the knowledge in the two professions.
- This plays out both in the attitudes of faculty and through student expectations.

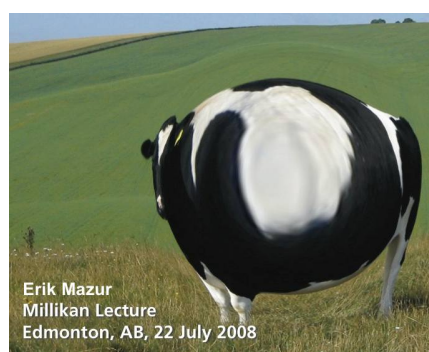
NEXUS Workshop

1/11/12

## + Thinking about teaching physics...

18

- “Physics should be as simple as possible – but not simpler.” (Einstein)
- “The physics we are learning in this class is simple – but seeing that it is simple can be exceedingly difficult.” (Redish)



Erik Mazur  
Millikan Lecture  
Edmonton, AB, 22 July 2008

NEXUS Workshop

## + Physics simplicity

19

- In physics we tend to seek the simplest example and understand it thoroughly.
- This serves as the “stake in the ground” to use to organize our later thinking about more complex systems.
- We work from a few clear (nearly) universal principles and tie everything to them – often deriving dozens of equations from one starting point – plus assumptions about situations and initial conditions.

NEXUS Workshop

1/11/12

## + Physics tries to establish a simple core

20

- Although there is a lot to do to interpret all this, it both
  - ties to everyday experience and
  - has a small number of ideas and relations that organize the complexity of the material.
- Bio looks different.

NEXUS Workshop

1/11/12

## + Intro Physics teaching

21

- Intro physics often stresses reasoning from a few fundamental principles.
- Physicists often stress building a complete understanding of the simplest possible examples – and often don't go beyond them at the introductory level.
- Physicists often quantify their view of the physical world and model with math.
- Physicists think with equations.
- Introductory physics typically restricts itself to the macroscopic level and almost never considers chemical energy

NEXUS Workshop

1/11/12

## + Thinking about teaching biology...

22

- Biology seems irreducibly complex.
- Every living organism involves thousands of chemicals and tens of thousands of reactions. (More? OK)
- Biology has a “historicity” that physics lacks. It could, in principle, have happened some other way.

NEXUS Workshop

1/11/12

## + Problem

23

- Biological systems larger than the simplest viruses have complex structures at a multiplicity of levels.
- Even beginning to talk about a complex biological system requires learning a large new (and arbitrary) vocabulary.

Technical words in intro physics are often based on terms in everyday speech. While this causes some confusion, it also gives an interpretive foothold.

NEXUS Workshop

1/11/12

## + Intro Biology teaching

24

- By its very choice of subject biology is complex.
- Most introductory biology is qualitative.
- Biology is fundamentally historical.
- Much of introductory biology is descriptive (and introduces a large vocabulary) though
- Biology – even at the introductory level – looks for mechanism and frequently considers micro to macro connections.
- Chemistry is much more important to intro bio than physics (or math).

NEXUS Workshop

1/11/12

## + Meta-goal

25

- To create a course that both physicists and biologists will see as authentic to their discipline AND
- that students will see as giving them insight into biology that is important to them in their vision of their future selves as scientists.

NEXUS Workshop

1/11/12

## + What we've done so far

26

- Begun a fine-grained analysis of competencies.
- A wikibook for student readings
  - Students read 2-3 webpages before each class and write a brief summary and question for each.
- Homework problems that do physics skill development in biological contexts.
- In-class clicker problems and group problems
- Observations
  - Video of lectures (students discussions and whiteboard work)
  - Interviews with individual students in physics and bio classes

NEXUS Workshop

1/11/12

## + Questions for discussion: 1

27

### ■ Content?

- Can we create materials that are flexible enough that they can meet the needs of different groups?
  - Research biology (mostly micro)
  - Research biology (mostly macro)
  - Medical (mostly clinical)
  - Medical (instrumentation)

### ■ Contributors?

- Many of us are working on this with many different orientations. Can we combine and support each other instead of competing?

NEXUS Workshop

1/11/12

## + Questions for discussion: 2

28

### ■ Competencies?

- Can we identify competencies at a fine enough grain size that we can see how to create materials to help teach them?
- Can such an identification help us to develop standardized assessment instruments to measure competencies?

NEXUS Workshop

1/11/12

+

## Preliminary results

1/11/12

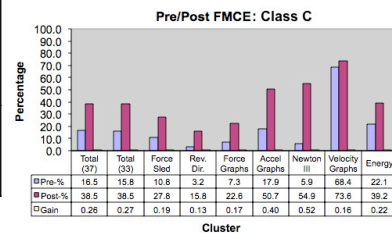
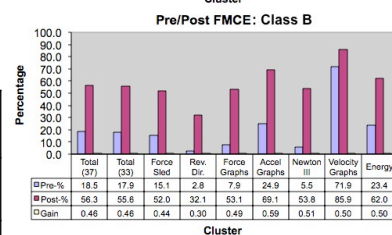
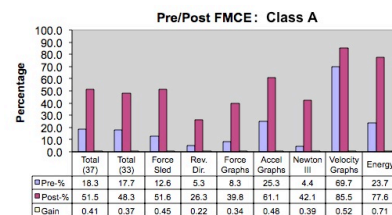
NEXUS Workshop

29

**+ If you suppress traditional mechanics a bit and stress energy instead, what happens?**

		N	$\langle g_{KF} \rangle$	$\langle g_E \rangle$
<b>A</b>	NEXUS test class	20	0.41	0.71
<b>B</b>	Reformed traditional (Epistemologized / with tut.)	189	0.46	0.50
<b>C</b>	Traditional (with tut.)	201	0.26	0.22

NEXUS Workshop



# Working content Physics 131

- \* Needs some revision
- \*\* First draft. Needs significant revision
- \*\*\* Planned but not yet written

## Overview

### Introduction to the class

- The disciplines: Physics, Biology, Chemistry, and Math \*
- Science as making models \*\*
- Phenomenology and mechanism
- Reductionism and emergence \*\*\*
- What Physics can do for Biologists \*

### Thinking about Thinking and Knowing

- The nature of scientific knowledge
- Models of memory
  - An evolutionary model of memory
  - Working memory and long term memory \*\*\*
  - Making meaning: association \*\*\*
  - Selective attention
- Tools for knowing (epistemological icons)
  - Choosing a channel on cat television (limiting the scope of exploration)
  - Shopping for ideas
  - Choosing foothold ideas
  - Playing the implications game
  - Seeking coherence (building a safety net)
  - Sense making
  - Refining intuitions
  - Representation translation
  - Dangerous bend

### Modeling with mathematics

#### Using mathematics in science

- How math in science is different from math in math
- Measurement
- Dimensions and units
  - Complex dimensions and dimensional analysis
  - Changing units
  - Natural scales
  - Considering change

- The parameters of matter
- Multiple symbols: What's a variable?
- Estimation
  - Scales in biology
  - Useful numbers

### Mathematics Recap

- Significant (and insignificant) figures
- Scientific notation
- The idea of algebra: unknowns and relationships \*\*\*
- Functions and functional dependence
  - Scaling \*
  - Powers and exponents
    - Log-log plots
- Values, change, and rates of change
  - Derivatives
    - What is a derivative, anyway?
  - Integrals
    - What do I actually have to know about integrals?
- Approximations \*\*\*
- Trigonometry
- Probability
  - Why probability matters

## Kinematics

### The Main Question: How do things move?

#### Where and When?

- Coordinates
- Vectors
  - Multiplying vectors
    - The dot product
    - The cross product
  - The gradient -- a vector derivative
- Time
- Graphs

#### Kinematic Variables

- Velocity
  - Average velocity
  - Instantaneous velocity

- [Calculating with average velocity](#)
- [Acceleration](#)
  - [Average acceleration](#)
  - [Instantaneous acceleration](#)
  - [Calculating with acceleration](#)

## Laws of Motion

### [Newton's Laws](#)

- [Physical content of Newton's Laws](#)
  - [Object egotism](#): Objects respond to the interactions they feel at the instant they feel them
  - [Inertia](#): Interactions change velocities
  - [Interactions](#):
  - [Superposition](#): The effects of interactions add like vectors
  - [Mass](#): Interactions are shared over parts of an object
  - [Reciprocity](#): When two objects interact, the forces they each exert forces on each other
- [Formulation of Newton's Laws as foothold principles](#)
  - [Quantifying impulse and force](#)
    - [On the definition of force](#)
    - [Kinds of forces](#)
    - Adding forces
  - [Newton 0](#)
    - [Free-body diagrams](#)
  - Newton's 1st law
  - [Newton's 2<sup>nd</sup> law](#)
    - [Reading the content in Newton's 2<sup>nd</sup> law](#)
    - [Newton 2 as a stepping rule](#)
      - [Newton 2 on a spreadsheet](#)
  - [Newton's 3rd law](#)

### [Kinds of Forces](#)

- [Springs](#)
  - [Realistic springs](#)
  - [Normal forces](#)
  - [Young's modulus](#)
  - [Tension forces](#)
- [Resistive forces](#)

- [Friction](#)
  - [Viscosity](#)
  - [Drag](#)
  - [Reynold's number](#)
- [Gravitational forces](#)
  - [Flat-earth gravity](#)
    - [Free-fall in flat-earth gravity](#)
    - [The effect of air resistance \\*\\*\\*](#)
  - [Universal gravitation \\*\\*\\*](#)
    - The gravitational field \*\*\*
- [Electric forces](#)
  - [Charge and the structure of matter](#)
  - [Polarization](#)
  - [Coulomb's law](#)
    - [Coulomb's law -- vector character](#)
    - [Reading the content in Coulomb's law](#)
  - [The Electric field](#)

### [Linear momentum](#)

- [Restating Newton's 2nd law: momentum](#)
- [Momentum conservation](#)

## Macro models

### [Solids](#)

- [Density-solids](#)
- [Young's modulus](#)
- [Breaking strain](#)
- [Bulk modulus -- solids](#)
- [Shear modulus](#)

### [Fluids](#)

- [Pressure](#)
- [Archimedes' Principle](#)
- [Buoyancy](#)
- Gases
  - Partial pressures
  - Bulk modulus -- gases
- Liquids
  - [Bulk modulus --liquids](#)
  - [Internal cohesion](#)
    - [Surface tension](#)
    - [Viscosity](#)

- [Fluid flow](#)
  - [Quantifying fluid flow](#)
  - [The continuity equation](#)
  - [Internal flow -- the HP equation](#)
    - [Internal flow -- the HP Equation \(advanced\)](#)
  - [External flow -- lift and drag](#)

### [Heat and temperature](#)

- [Thermal properties of matter](#)
  - [Thermal energy and specific heat](#)
  - [Heat capacity](#)
  - [Heat transfer](#)
    - [Biology of heat transfer](#)

### [Energy: The Quantity of Motion](#)

- [Kinetic energy and the work-energy theorem](#)
  - [Reading the content in the Work-Energy theorem](#)
- [Energy of place -- potential energy](#)
  - [Gravitational potential energy](#)
  - [Spring potential energy](#)
  - [Electric potential energy](#)
- [The conservation of mechanical energy](#)
  - [Interpreting mechanical energy graphs](#)
  - [Forces from potential energy](#)
- [Energy losses -- thermal energy](#)\*\*\*
- [An essential energy storage for life -- chemical energy](#)\*\*\*
  - [Energy at the sub-molecular level](#)
  - [Atomic and Molecular forces](#)
    - [Interatomic forces](#)
      - [The Lennard-Jones Potential](#)
    - [Chemical bonding](#)
    - [Hydrogen bonding](#)
  - [Chemical energy](#)

### [The Micro to Macro Connection](#)

#### [Thermodynamics and Statistical Physics](#)

- [Kinetic theory: the ideal gas law](#)
- [The role of randomness: Biological implications](#)
- [Diffusion and random walks](#)
  - [Fick's law](#)
- The 0<sup>th</sup> law of thermodynamics
  - [Equipartition](#)
- [The 1<sup>st</sup> law of thermodynamics](#)
- The 2<sup>nd</sup> law of thermodynamics
  - [Why we need a Second Law of Thermodynamics](#)
  - [The 2<sup>nd</sup> Law of Thermodynamics: A Probabilistic Law](#)
  - [Implications of the Second Law of Thermodynamics](#)

# umdberg: Worm Problem:How big is a worm - P: Solution

## Working content

The earthworm absorbs oxygen directly through its skin. The worm does have a good circulatory system (with multiple small hearts) that brings the oxygen to all the cells. But the cells are distributed through the worm's volume and the oxygen only gets to come in through the skin -- so the surface to volume ratio plays an important role. Let's see how this works. Here are the worm's parameters.

A typical specimen of the common earthworm (*Lumbricus terrestris*) has the following average dimensions:

- Mass ( $m$ ) - 3.7 g
- Length ( $L$ ) - 12 cm
- Width ( $2R$ ) - 0.64 cm

The skin of the worm can absorb oxygen at a rate of  $A = 0.24 \mu\text{mole}$  ( $\mu\text{mole} = 10^{-6}$  moles) per square cm per hour.

The body of the worm needs to use approximately  $B = 0.98 \mu\text{mole}$  ( $\mu\text{mole} = 10^{-6}$  moles) of oxygen per gram of worm per hour.



SpeedBump by Dave Coverly: with permission

A. It is reasonable to model the shape of the earthworm as a solid cylinder. Using the dimensions of a typical earthworm above, calculate its surface area (ignore the surface areas of the blunt ends in all calculations), volume, and density.

$$\text{Surface area} = 2\pi RL = (2\pi)(0.32 \text{ cm})(12 \text{ cm}) = 24 \text{ cm}^2$$

$$\text{Volume} = \pi R^2 L = (\pi)(0.32 \text{ cm})^2 (12 \text{ cm}) = 3.9 \text{ cm}^3$$

$$\text{Density} = d = \text{Mass/Volume} = (3.7 \text{ g}) / (3.9 \text{ cm}^3) = 0.95 \text{ g/cm}^3$$

B. If the worm is much longer than it is wide ( $L \gg R$ ) is it OK to ignore the end caps of the cylinder in calculating the area? How does the surface area and volume of the worm depend on the length of the worm,  $L$ , and the radius of the worm,  $R$ ?

The surface area of the worm modeled as a cylinder is  $S = 2\pi RL + 2\pi R^2$  with the last term being the area of the two end caps.

The ratio of the area of the endcaps to the "wrap around" part of the cylinder is  $2\pi R^2 / 2\pi RL = R/L$ . If  $L \gg R$  this is OK to ignore.

If we drop the endcaps, the surface area increases linearly with  $L$  and  $R$ .

The volume of the worm modeled as a cylinder is  $\pi R^2 L$  so it increases linearly with  $L$  and quadratically with  $R$ .

C. For an arbitrary worm of length  $L$ , radius  $R$ , and density  $d$ , write an equation (using the symbols  $A$  and or  $B$  rather than the numbers) that expresses the number of moles of oxygen the worm absorbs per hour and the number of moles the worm uses per hour. What is the condition that the worm takes in oxygen at a rate fast enough to survive? Can the typical worm described above absorb sufficient oxygen to survive?

$$\text{oxygen absorbed per hour} = \text{area} \times (\text{rate of absorption/area}) = 2\pi RL A$$

$$\text{oxygen used per hour} = \text{grams} \times (\text{rate of use/gram}) = \pi R^2 L B$$

$$\text{condition is that absorbed oxygen must be more than oxygen used: } 2\pi RL A > d\pi R^2 L B \text{ or } 2A > dRB$$

$$2A/dB > R$$

The combination  $2A/dB$  has the value

$$\frac{2A}{dB} = \frac{2 \left( 0.24 \frac{\mu\text{mole}}{\text{cm}^2 \text{h}} \right)}{\left( 0.95 \frac{\text{g}}{\text{cm}^3} \right) \left( 1 \frac{\mu\text{mole}}{\text{g h}} \right)} = \left( \frac{2 \times 0.24}{0.95} \right) \frac{\mu\text{mole cm}^3 \text{ g h}}{\mu\text{mole g cm}^2 \text{ h}} = 0.50 \text{ cm}$$

Since the worm has a radius of 0.32 cm, less than 0.50 it can survive.

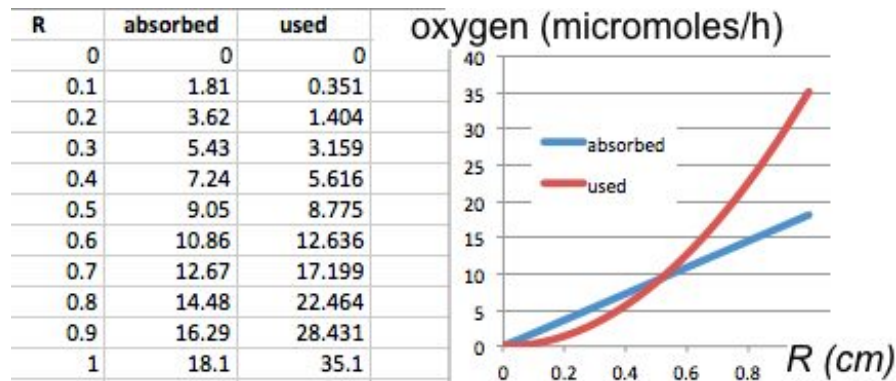
D.1. Consider the effect of changing the various size parameters of a worm. First consider a worm of length 12 cm that grows by keeping its length the same but increasing its radius. Use a spreadsheet to plot the total oxygen absorbed through the skin of the worm and the total oxygen used by the worm as a function of its length from a radius of 0 cm (not really reasonable) up to a radius of 1 meter. Do the two curves cross? Explain what the crossing means and what its implications are.

oxygen absorbed by this worm as a function of radius

$$= (2\pi L A) R = (6.28)(12 \text{ cm})(0.24 \mu\text{mole}/\text{cm}^2\text{-h}) R = (18.1 \mu\text{mole}/\text{cm-h}) R$$

oxygen used by this worm as a function of radius

$$= d\pi R^2 L B = (d\pi L B) R^2 = (0.95 \text{ g}/\text{cm}^3)(3.14)(12 \text{ cm})(0.98 \mu\text{mole}/\text{g-h}) R^2 = (35.1 \mu\text{mole}/\text{cm}^2\text{-h}) R^2$$



For small radii there is more oxygen absorbed than used. But the oxygen used grows as a function of radius faster than it can be absorbed. Past 0.5 cm radius it can't absorb enough oxygen. The worm will die.

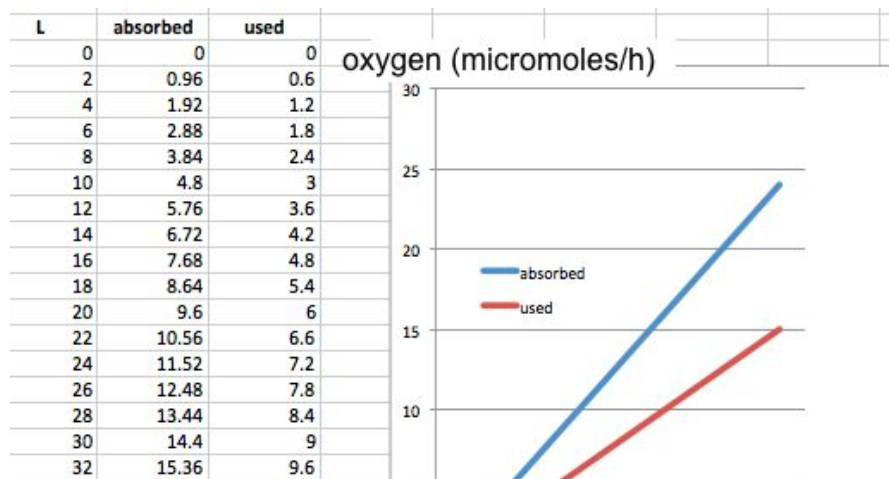
D.2. Now consider a worm of width 0.64 cm (radius 0.32 cm) that grows by keeping its width the same but increasing its length. Use a spreadsheet to plot the total oxygen absorbed through the skin of the worm and the total oxygen used by the worm as a function of its length from a length of 0 cm (not really reasonable) up to a length of 50 cm. Do the two curves cross? Explain what the crossing means and what its implications are.

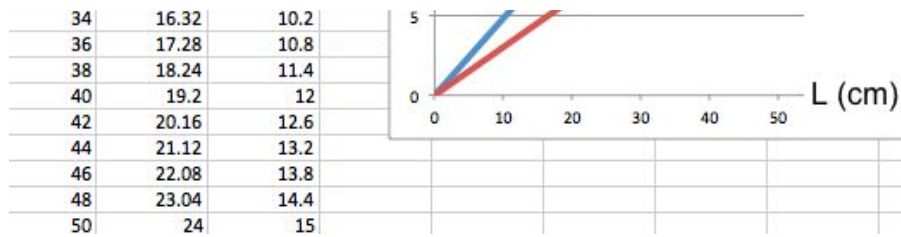
oxygen absorbed by this worm as a function of length

$$= (2\pi R A) L = (6.28)(0.32 \text{ cm})(0.24 \mu\text{mole}/\text{cm}^2\text{-h}) L = (0.48 \mu\text{mole}/\text{cm-h}) L$$

oxygen used by this worm as a function of length

$$= d\pi R^2 L B = (d\pi R^2 B) L = (0.95 \text{ g}/\text{cm}^3)(3.14)(0.32 \text{ cm})^2(0.98 \mu\text{mole}/\text{g-h}) L = (0.30 \mu\text{mole}/\text{cm}^2\text{-h}) L$$





For this radius worm, the amount of oxygen absorbed is always greater than the amount used.  
There is no limitation on the length.

D.3. Write (in symbols) an equation that represents the crossover condition -- that the oxygen taken in per hour exactly equals the oxygen used per hour. Cancel common factors. Discuss how this equation tells you about what you learned about worm growth by doing the two graphs.

$$2A/dB = R$$

This equation tells you the radius of the largest worm that can live by absorbing oxygen through its skin. Since it does not depend on the length it tells you that the length does not affect whether it can survive by absorption or not.

E. Our analysis in D was a modeling analysis. An organism like an earthworm might grow in two ways: by just getting longer or isometrically -- by scaling up all its dimensions. What can you say about the growth of an earthworm by these two methods as a result of your analysis in part D? Does a worm have a maximum size? If so, in what sense? If so, find it.

If it just gets longer it can keep going as long as it can hold together. Oxygen absorption is not a problem for it. But if it begins to grow isometrically, the radius gets bigger too and when it gets larger than 0.5 cm (a width of 1 cm), the worm will die.

F. In typical analyses of evolution and phylogenetic histories, earthworm-like organisms are the ancestors of much larger organisms than the limit here permits. Discuss what sort of variations in the structure of an earthworm might lead to an organism that solves the problem of growing isometrically larger than the limit provided by this simple model.

Various answers are possible, but the structure of the analysis above gives us clues. As the organism gets bigger, more oxygen needs to be brought in than a cylindrical structure would allow, so one way to get around it is to change away from a cylindrical structure. Beginning to create "pockets" in the skin creates more surface area without increasing the volume. If those pockets get bigger the organism can begin to evolve a fractal like structure in which lots of oxygen can be brought in - a gill or a lung. Another way to improve the ratio is to develop new chemistry that either improves the rate of oxygen absorption through the skin (increase  $A$ ) or decrease the metabolic rate of burning of oxygen throughout the volume (decrease  $B$ ).

# umdberg: Kinematics:Moving through a cell -- Solution

## Moving through a cell

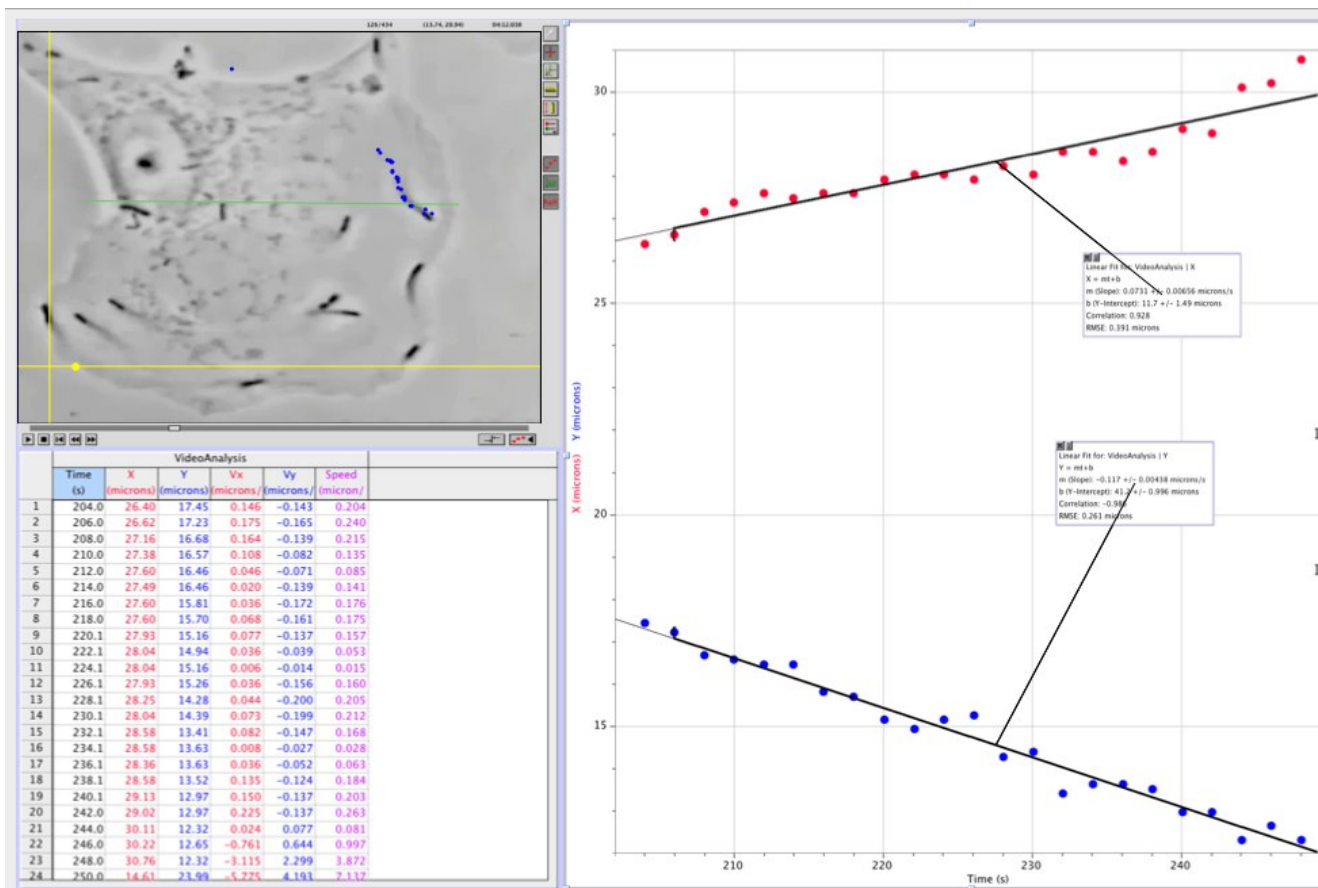
We will study the motion of the bacteria in the video at the right using *LoggerPro*. You can view the full video (with commentary) for the infection of a cell by [listeria on YouTube](#). To carry out this assignment, you should have *LoggerPro* installed on your computer.

Watch the video through a few times and pick a bacterium that you choose to follow. Since the bacteria are moving in 3D and we are only seeing them projected in 2D, some of the time they might be moving up and down -- perpendicular to the screen. And since the mechanism is random rather than continuous, the motion may be a bit herky-jerky. Pick one bacterium to follow that seems to be moving in a straight line.

A. Take data from the video by first picking an origin and coordinate axes and then setting the distance scale. The width of a typical mammalian cell is approximately 30 microns across. The video frames are taken every 2 s (and the cmbl is set up for that time step). The graph is set up to plot x vs t and y vs t.

From your data, use the curve fitting to fit a straight line to each of the graphs you generated. Read off the x and y components of the velocity from the fit and construct the average speed of the bacterium you have chosen.

Here's the result with the fit. In order to get the scales right, I changed the scale to make the cell 30 microns in diameter. After collecting the data, I changed the frame rate (on the Options/Movie Options dialog box) to 0.5 frames second. (You can't do this before you take the data or it won't let you take data. This is a bug in the program.)



The fit to the x-position gives the result

$$x(t) = (0.07 \text{ microns/s})t + 11.7 \text{ microns}$$

and the fit to the y-position gives the result

$$y(t) = (-0.12 \text{ microns/s}) t + 41.2 \text{ microns.}$$

The slopes are the velocities so the result is

$$v_x = 0.05 \text{ microns/s}$$

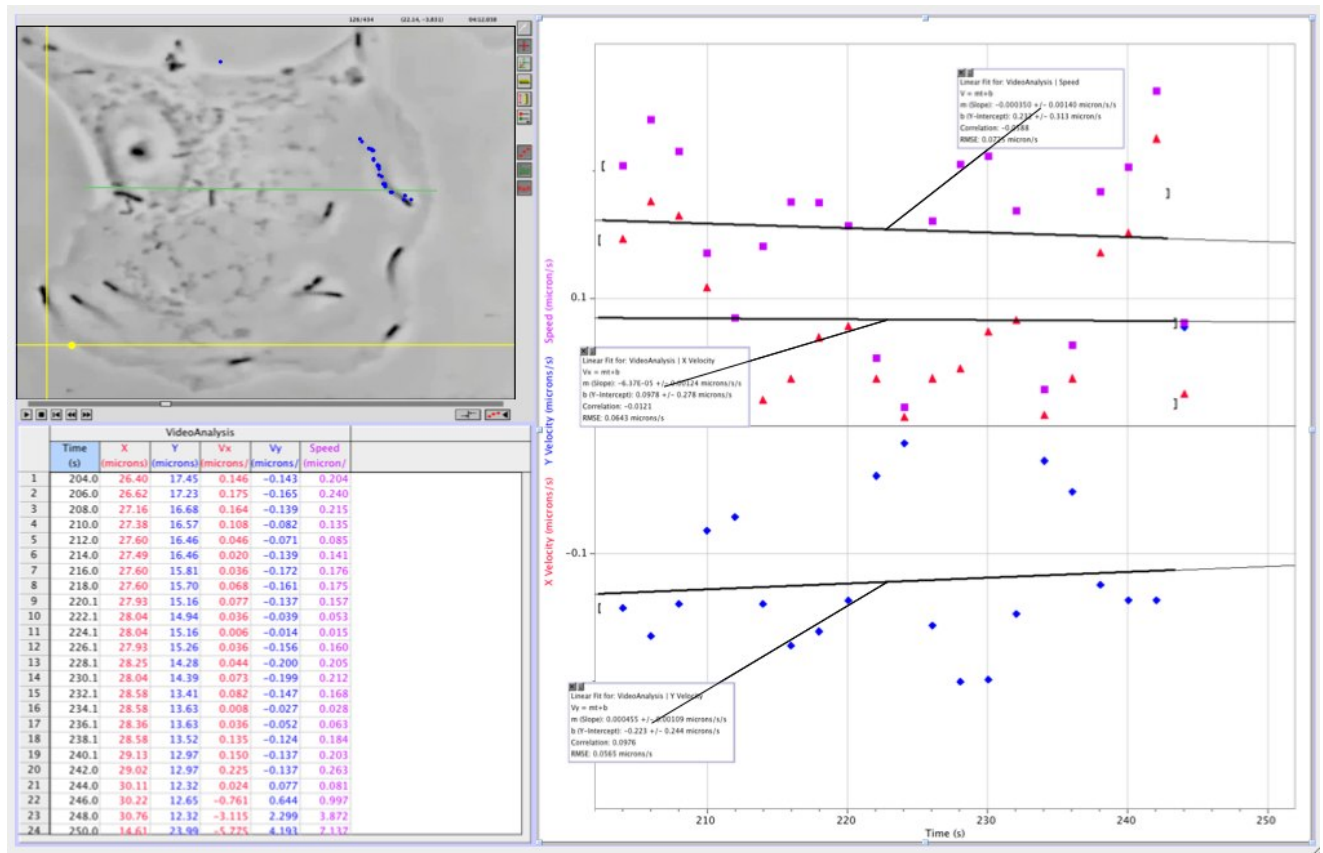
$$v_y = -0.12 \text{ microns/s}$$

$$v = \text{Sqrt}(v_x^2 + v_y^2) = 0.13 \text{ microns/s (speed).}$$

If I hadn't changed the scale for time in the program, I would have had to multiply the velocity I got out of the graph by something. Since the program calculates (if you don't change anything) assuming 30 frames/s and we have 1/2 frame/s, the delta t it uses to calculate velocity is 60 times too small. This will make the velocity shown 60 times too big. So to get the right value you have to divide by a factor of 60.

B. Now click on the x-y symbols on the left of the graph's vertical axis and choose "More". Turn off the x and y plotting and turn on  $v_x$ ,  $v_y$ , and  $v$  (velocity components and speed). Fitting these each with a straight line, which of the parameters of your straight line fit should match what you found in part A? Why? How good is the agreement between the two methods? Which one do you trust most and why?

Here's what I get for the velocity and speed plots and fits.



The numbers are almost constants. We just want the constant value ignoring any change in the velocity. (This is different from looking at the position graph, where the velocity we wanted was the slope -- the coefficient of t.)

The x velocity is  $v_x = 0.10 \text{ micron/s}$ ; the y velocity is  $v_y = -0.22 \text{ micron/s}$  and the speed is  $v = 0.21 \text{ micron/s}$ . These are a bit bigger than the values we got from fitting the x curves -- mostly because the velocity points scatter all over the place. The motion is in "fits and starts". I would guess that the average velocity obtained from the smoothing of the position curve is more reliable to work with to get an average since there is so much fluctuation in the velocity values.

C. Now let's take our best speed and use it to give information about the actin polymerization taking place. There is evidence that many actin chains are attached to the "actin comet tail" connected to the bacterium. But for now, let's model this as just a single chain with actin being added. From your data on the speed at which the bacterium moves and [the size of an actin molecule](#) (diameter about 5.4 nm), estimate the number of actins being added to the chain per second.

If we are moving at a speed of about 0.13 microns/s (from part A), we have to add N actin molecules per second with  $N (5.4 \text{ nm}) = 0.13 \text{ microns/s}$ .

Converting,  $1 \text{ nm} = 10^{-9} \text{ m}$  and  $1 \text{ micron} = 10^{-6} \text{ m}$ . So  $1 \text{ nm} = 10^{-3} \text{ microns}$ . So our calculation is (multiplying by an

appropriate form of "1" to change units)

$$N = (0.13 \text{ microns/s}) \times (1/ 5.4 \text{ nm}) = (0.13/5.4) \times (\text{micron/nm-s}) \times (1 \text{ nm} /10^{-3} \text{ micron}) = 24 \text{ actins/s}$$

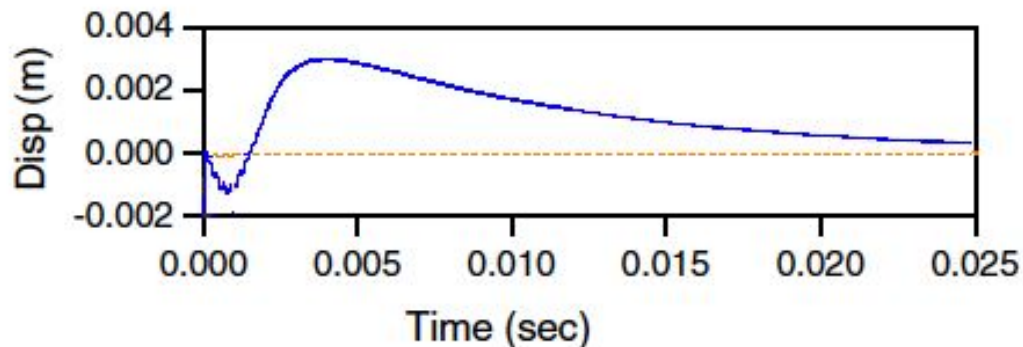
So about 20-30 actin molecules added per second.

Joe Redish 9/16/11

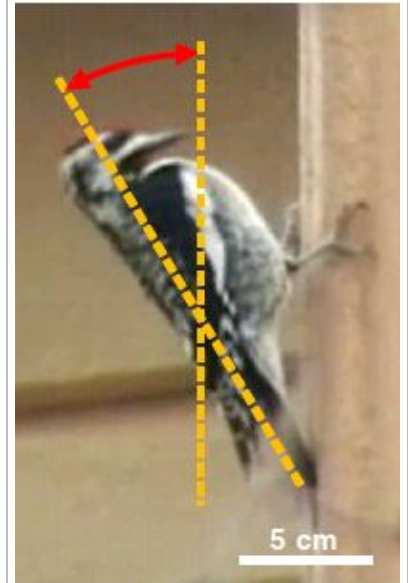
# umdberg: Newtons Laws (open):Force on a Woodpecker-- Solution

Woodpecker's skulls are adapted to be able to deliver a large force to a wood surface in order to drill a hole to get at insects under the bark of the tree. In order to get an idea of the scale of the forces involved we will consider a model from a 2011 paper by Yoon and Park.\* A photo from their paper is shown at the right to set a scale.

A. In the figure below is shown Yoon & Parks model of the beak displacement.



From the displacement curve of the tip of the beak, we can infer the force that the woodpecker's head and beak feel -- and therefore the forces they exert on the tree.



Yoon & Park

(This curve has been smoothed somewhat, especially in the initial ms when the woodpecker feels a high frequency vibration. Although this has important consequences for the woodpecker's brain mounting structure, we will ignore it here.)

A. The displacement in the graph is measured from the surface of the wood. From the data in the graph, estimate the speed with which the beak tip hits the wood and the acceleration that turns its motion around. (This all occurs in the first millisecond.)

The beak goes from 0.000 m to about -0.001 m (about 1 mm) in a time which looks to be about 0.001 s and back in about the same time. So the initial velocity going into the wood is about

$$v = \Delta x / \Delta t = (-0.001 \text{ m}) / (0.001 \text{ s}) = -1 \text{ m/s}$$

Going back out it goes about the same distance in the same time so the final velocity coming out of the wood is +1 m/s. This makes the change in the velocity equal to

$$v_f - v_i = (1 \text{ m/s}) - (-1 \text{ m/s}) = 2 \text{ m/s}$$

This take place in about 1/5 of the time to go down, so perhaps 0.0002 s. This makes the acceleration equal to

$$a = \Delta v / \Delta t = (2 \text{ m/s}) / (2 \times 10^{-4} \text{ s}) = 10^4 \text{ m/s}^2$$

or about 10,000 m/s<sup>2</sup>.

B. Using the  $F\Delta t = m\Delta v$  form of Newton's second law, infer the magnitude of the force on the woodpecker's beak and the acceleration of the beak. (The acceleration is frequently expressed in units of  $g \sim 10 \text{ m/s}^2$ . Express your acceleration in  $\text{m/s}^2$  and in "g"s.) Estimate anything you need to know that isn't given in the data and explain your estimate.

The force then =  $m \Delta v / \Delta t$ . We have just calculate the ratio of the changes (the acceleration) so all we have to do is estimate the mass of the woodpecker's head. From the scale shown on the figure, the head looks to be about 3 cm in diameter or 1.5 cm in radius. Let's model it as a sphere of radius 1.5 cm. The volume of a sphere is  $4\pi r^3/3$ , so

$$\text{volume} \sim 4 (1.5 \text{ cm})^3 \sim 15 \text{ cm}^3 \sim 15 \times 10^{-6} \text{ m}^3.$$

The head should be about the same density as water -- 1000 kg/m<sup>3</sup>. So that would give a mass of

$$\text{mass} \sim (10^3 \text{ kg/m}^3) \times (15 \times 10^{-6} \text{ m}^3) = 15 \times 10^{-3} \text{ kg} (= 15 \text{ g})$$

So the force is

$$\text{force} = (1.5 \times 10^{-2} \text{ kg}) \times (10^4 \text{ m/s}^2) \sim 150 \text{ N}.$$

That's about 30 pounds!

C. How does the force that the tree exerts on the woodpecker's beak (as shown here) compare to the force the beak exerts on the tree? Explain your reasoning.

The forces are equal and opposite in direction by Newton's 3<sup>rd</sup> law.



\* S-H Yoon & S. Park, A mechanical analysis of woodpecker drumming and its application to shock absorbing systems, Bioinspiration & Biomimetics, 6 (2011) 016003 (12 pp).

# umdborg: Kinematics (open):The cat and the antelope

One of the classic examples of predator-prey is the big cat and the herbivore. Like many predators and their prey, natural selection leads them on an "arms race", with the faster variants on both sides having a survival advantage, leading to both the prey and the predator being able to run at very high speeds. The actual interaction between predator and prey is quite complex, involving not just speed, but the evolution of stamina, jumping, pouncing, and camouflage.\* For this problem, let's just model one aspect of the predator prey interaction by considering a simple question. How far away is safe enough?

Fast herbivores that flee fast predators like to avoid locations where a camouflaged predator can hide. How far from a dangerous area should a prey animal stay in order to be able to escape if a predator bursts into view? This distance depends on how the animal runs. Let's consider a specific example.

A cheetah is one of the fastest animals, but it can only maintain its high speed for a short time. The Thomson's gazelle can continue to run at a steady pace for a long time, but its maximum speed is less than that of the cheetah. Here are some of the parameters of their motions.

<p>Thomson's gazelle:</p> <ul style="list-style-type: none"><li>• Acceleration: from 0 to 90 km/hr in about 18 seconds.</li><li>• Max speed: 90 km/hr</li><li>• Stamina: Can maintain its maximum speed for long periods of time (compared to the cheetah).</li></ul>	 <p>Photo by David Bygott, CC permission</p>
<p>Cheetah:</p> <ul style="list-style-type: none"><li>• Acceleration: from 0 to 120 km/hr in 3 seconds</li><li>• Max speed: 120 km/hr but can only maintain it for about 30 seconds.</li><li>• After its initial high speed sprint, it collapses due to heat exhaustion -- its temperature may rise as high as 105° F -- and itself becomes vulnerable to larger predators.</li></ul>	 <p>Photo by Jason Bechtel CC permission</p>

A. Let's begin by considering a very simple model of predator-prey interaction. Let's assume that the predator creeps up on a resting prey to a distance,  $d$ , and catches the prey during the period when they are both accelerating. At time  $t = 0$ , the prey sees the predator and both take off, accelerating as fast as they can.

A.1 The cheetah only accelerates for about 3 seconds. If they both start accelerating at the same time, what will the gazelle's and cheetah's average velocity be at the end of that time interval?

A.2 How far will they each have traveled in that time interval? How far away does the gazelle have to start in order for the cheetah not to catch it during its acceleration phase?

B. Now let's refine the model by including the cheetah's high speed sprint as well as its acceleration. We notice that the cheetah can run faster than the gazelle, but only for 30 seconds. If they are both running straight, how far away must the gazelle be from the cheetah in order that the cheetah not catch it before the cheetah collapses from exhaustion?

C. Find a symbolic representation of the maximum distance,  $d$ , for which a general predator can catch its prey if:

- The predator can accelerate at a rate of  $a_1$  for a time  $t_1$  and can run at that final speed for a time  $T_1$ . After that time, it must stop running.
- The prey can accelerate at a rate of  $a_2$  for a time  $t_2$  and can run at that final speed for a long time -- at least for a lot longer than the predator can run.

Check your final answer by seeing if the values you got for part B are obtained by putting the appropriate parameters into your equation. Consider various limiting cases to see if your equation makes sense.

D. The equations you have generated for part C have been turned into a spreadsheet simulation that creates a graph showing the position of the cheetah and the gazelle as a function of time given values for the parameters  $a_1$ ,  $t_1$ ,  $T_1$ ,  $a_2$ ,  $t_2$ , and  $d$ . Download the file cheetahgazelle.xls and explore the dependence of the resulting paths on one of the parameters. Describe in words something interesting that you have learned about the chase from this exploration and illustrate your conclusion with printouts of a few graphs.

\* R, McNeill Alexander, *Principles of Animal Locomotion* (Princeton University Press, 2003), Chapter 1.

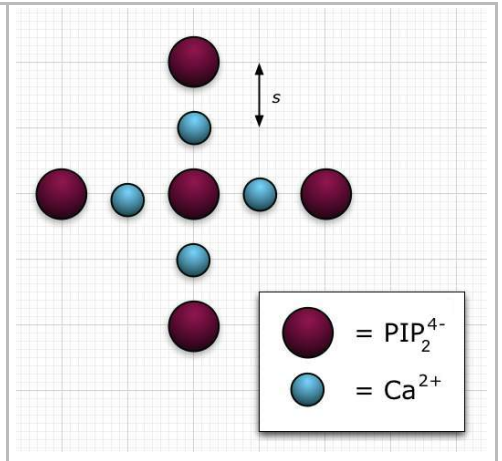
Joe Redish 8/19/11

# umdborg: Electricity: PIP<sub>2</sub> cluster stability (Solution)

The phospholipid molecule PIP<sub>2</sub> (phosphatidylinositol 4,5-bisphosphate) is an important constituent of eukaryotic cell membranes. Its hydrophilic head group has a greater negative charge (typically  $-4e$  under physiological pH) than most of the other phospholipids present in eukaryotic cell membranes. PIP<sub>2</sub> makes up only a small fraction of the membrane (typically  $\sim 1\%$  by mole fraction), but in spite of this low concentration, it is known to form clusters with multiple PIP<sub>2</sub> molecules that are thought to be important in cell signaling. It is an area of active research to understand the basis of this cluster formation.

Recent research\* (Wang, Janmey, and coworkers) provides support for an electrostatic mechanism for this clustering, in which  $\text{Ca}^{2+}$  ions provide an attractive interaction holding together the PIP<sub>2</sub> molecules. The detailed structure of how  $\text{Ca}^{2+}$  ions and PIP<sub>2</sub> molecules are arranged in these clusters is not yet known. In this problem we consider a highly simplified model simply to give a feel for how these interactions work.

A simplified model of such a cluster consists of five PIP<sub>2</sub> molecules, each with a headgroup charge of  $-4e$ , and four  $\text{Ca}^{2+}$  ions. They are all placed on a grid where each grid square has sides of length  $s$ , as shown in the figure at the right.



In this problem we will explore whether the electric forces on the PIP<sub>2</sub> molecules in this model cluster tends to pull it together or blow it apart.

(a) What is the net electric force on the central PIP<sub>2</sub> molecule in the cluster coming from all the other charges in the cluster?

The net electric force on the central charge is 0. This is obvious from the fact that the other charges are all paired on opposite sides so whatever force is exerted by one charge, there is an equal strength charge on the opposite side at the same distance. The resulting forces will be equal and opposite.

(b) To find out whether the electric forces pull the PIP<sub>2</sub> molecules in the cluster together or blow them apart, we only have to consider one of the PIP<sub>2</sub> molecules, say the one at the very top. Explain why.

The system is symmetric. Whatever force we find on the top molecule -- and it will be either toward the center or away from it -- the same magnitude of force will act on all the other outside PIP<sub>2</sub> molecules.

(c) The total force on the top ion will look like a pure number times the expression  $k_C e^2/s^2$ . Explain why this is true. Your explanation could involve a consideration of each force or use dimensional analysis (or you might come up with something else).

The force between the top ion and any other charge will be given by Coulomb's law. This says that the magnitude of the force between any two charges in the diagram will be

$$F_{A \rightarrow B} = k_C Q_A Q_B / d^2$$

The charge on each of the ions is some number times  $e$ , and every distance is some number times the spacing  $s$ . We will have to be breaking up into components, but that just multiplies that magnitude by a  $\sin$  or  $\cos$ , a pure number without dimensions. Therefore, every component of every force looks like a pure number times the expression  $k_C e^2/s^2$ .

(d) Find the net force on the top ion exerted by the other ions by finding the number that multiplies the expression given in

part (c). What direction does it point? Does this force push the top ion toward the center (tending to hold the cluster together) or push it away (tending to blow the cluster apart)?

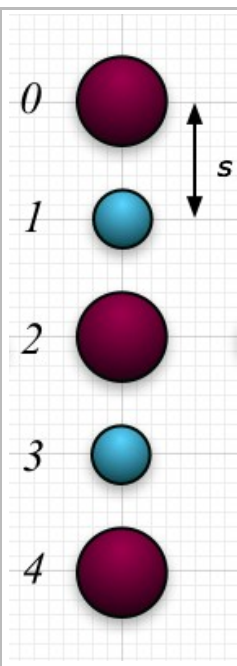
To actually get the force on the ion at the top is a bit of a mess. There are 8 other ions that are exerting an electric force on our charge at the top! We have to find all of these as vectors and add them up vectorially. This sounds hard, but let's take it a step at a time. Let's first do all the ones in a line below the charge we are interested in. The forces from these are all in the same direction so it's not so bad to add them up.

$$k_C e^2 / s^2.$$

Then we can find everything in terms of that.

Let's call our ion at the top, charge 0. And let's number the chain beneath it 1 through 4. It's then fairly clear that the force that ion 1 exerts on ion 0 is attractive (down). Since ion 1 has 2 units of charge and ion 0 has 4 units of charge, and since the distance is 1s, the force from 1 on 0 will be

$$F_{1 \rightarrow 0} = - [(2)(4)/(1)^2] F_0 = -8 F_0$$



Similarly, ion 2 has 4 units of charge and is a distance 2s away from ion 0, and is repulsive (up) so

$$F_{2 \rightarrow 0} = + [(4)(4)/(2)^2] F_0 = +4 F_0$$

We can easily find the other two to be

$$F_{3 \rightarrow 0} = - [(2)(4)/(3)^2] F_0 = -8/9 F_0$$

$$F_{4 \rightarrow 0} = + [(4)(4)/(4)^2] F_0 = +1 F_0$$

The overall result is the sum of these:

$$F_{1 \rightarrow 0} + F_{2 \rightarrow 0} + F_{3 \rightarrow 0} + F_{4 \rightarrow 0} = (-8 + 4 - 8/9 + 1) F_0 = -3.9 F_0.$$

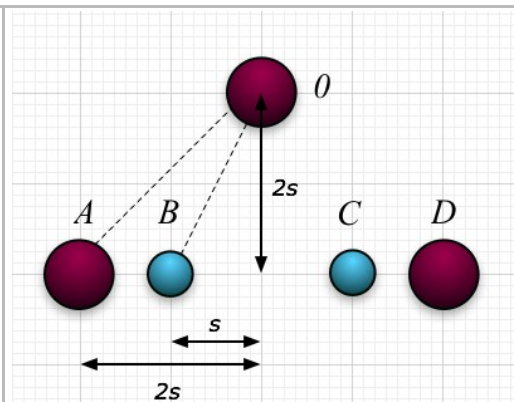
So the sum of all those in the chain below ion 0 is strongly down.

Now let's look at the result of the ones to the sides. We'll call them A, B, C, and D. A and D will pair to give a force upward while B and C will pair to give a force downward.

Let's first figure out the magnitude of the force of A on 0. (This will be the same as the magnitude of D on 0 but they will point in different directions.) How far is A from 0? It's along the hypotenuse of a right triangle whose sides are each 2s. Therefore the distance is the square root of  $(2s)^2$  plus  $(2s)^2$  or  $8s^2$ . So the distance between A and 0 is  $\sqrt{8}s$ . Each has a charge of 4 and the distance comes in squared, so compared to  $F_0$ , we get,

$$F_{A \rightarrow 0} = [(4)(4)/8] F_0 = 2F_0 = F_{D \rightarrow 0}$$

Note this is a magnitude. We'll have to deal with directions later.



Now how about B and C? How far away are they from ion 0? They are along the hypotenuse of a right triangle whose sides

are  $2s$  and  $1s$ . Therefore the distance is the square root of  $(2s)^2$  plus  $(1s)^2$  or  $5s^2$ . So the distance between B and 0 is  $\sqrt{5}s$ . B has charge  $-2$  so the magnitude of the force (compared to  $F_0$ ) is

$$F_{B \rightarrow 0} = [(2)(4)/5] F_0 = (1.6) F_0 = F_{C \rightarrow 0}$$

Again, these are magnitudes.

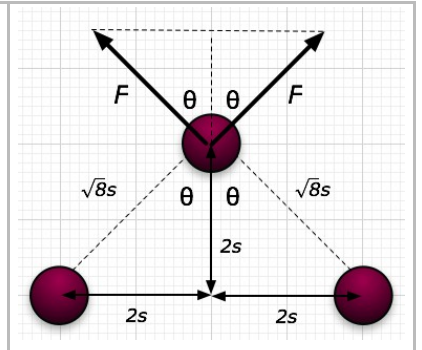
Now we have to get the directions.

The ions A and D exert forces on ion 0 as shown. Clearly the symmetry tells us that the horizontal components of these two forces will cancel; the vertical components will add. So if the magnitude of the forces are  $F$  then the contribution will be

$$2F \cos(\theta)$$

By similar triangles, we can see from the distances in the lower graph that

$$\cos(\theta) = (2s)/(\sqrt{8}s) = 2/\sqrt{8} = 0.71$$



So since we figured out above that  $F$  is  $2F_0$ , the contribution of these two ions is an upward force of

$$2(2F_0) \cos(\theta) = 2.8 F_0.$$

We can similarly figure out that the contribution of the two off-axis Ca ion will be (since now  $\cos$  will be  $2/\sqrt{5}$ )

$$2F \cos(\theta) = -2 (1.6 F_0) \times 2/\sqrt{5} = -2.9 F_0.$$

The total result, adding everything together, is

$$-3.9 F_0 + 2.8 F_0 - 2.9 F_0 = -4.0 F_0$$

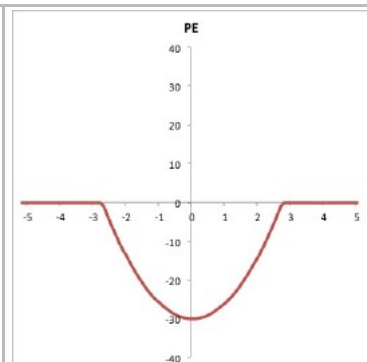
The total is negative -- down -- so the overall result is attractive! The cluster holds together.

\* Z. Li, et al., [Biophys. J., 97, 155-163](#) (2009).

Catherine Crouch, Julia Svoboda, and Joe Redish 11/19/11

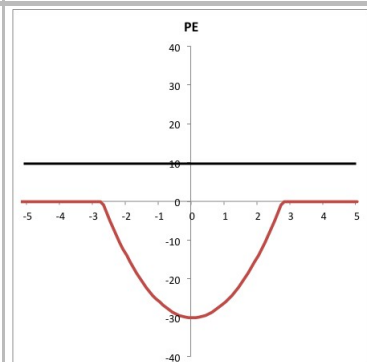
# umdberg: Energy (open):Bound states (solution)

One of the challenging ideas of using energy at the atomic and the molecular level is the idea of *bound states*. What this means is that you might start with two objects that have essentially zero energy kinetic energy, they get close and interact strongly in an attractive way. They find some way to emit energy into another form and wind up being stuck together -- bound. You have to put energy in -- do work on them -- in order to get them apart. Let's work through the language of potential energy to see how to talk about this.



A. Let's first talk about a simple problem that you have now had some practice with -- the motion of a skateboarder on a track. Suppose the track looks like a dip in the ground as shown in the figure. A potential shape like this is often referred to as a *potential well*, since it looks like a dug-out area for a well.

Now suppose that the skateboarder approaches the dip from the left traveling with a positive kinetic energy. The figure at the right shows the skateboarders total mechanical energy as a solid black line at a PE of 10 units (units unspecified).

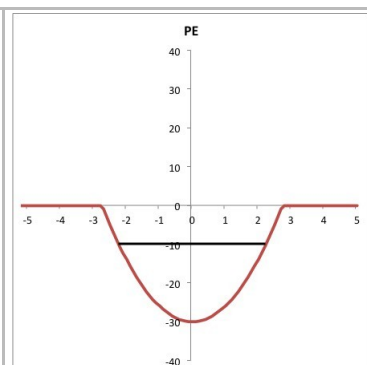


Describe the motion of the skateboarder and how her potential and kinetic energies change as she moves through the well.

She approaches the dip moving at a constant velocity. Her KE remains constant (and positive) and her PE remains 0. When she reaches the dip, her PE begins to become more negative. So to have her total energy constant, her KE becomes larger and she speeds up. She speeds up as the depth of the well increases, reaching a maximum speed at the well's bottom. As she begins to climb the right side of the well, the PE gets less negative so the KE gets less positive and she slows down. When she reaches the end of the well her KE is the same as it was before she entered so she continues on at the same speed she had before she reached the well.

B. Now suppose that she starts *inside the well* at a zero velocity -- say at point  $x = -2.5$  units with a total energy as shown by the heavy solid line.

Describe the motion of the skateboarder and how her potential and kinetic energies change as she moves through the well.



She starts at 0 velocity inside the well so with a 0 KE, a negative PE, and a negative total mechanical energy. The well slopes down to the right so there is a force pushing her to the right. She begins to accelerate in that direction and, as her KE increases, her PE gets more negative, keeping the total mechanical energy constant. She reaches her maximum speed at the bottom of the well and slows as she rises up on the right, coming to rest at the point  $x = +2.5$  units. Her energy is still negative so she can't leave the well.

C. Her total energy is shown in the figure as -10 units. How can this be? Is it reasonable for the total mechanical energy to be negative?

It is. The PE is really only relative. We can decide where we take it to be 0. If we choose it to be 0 when she is outside the well and at rest (a convenient place), then if her total energy is positive she can leave the well with a positive speed. If her total energy is negative it means that positive energy has to be added to bring her to a point at rest

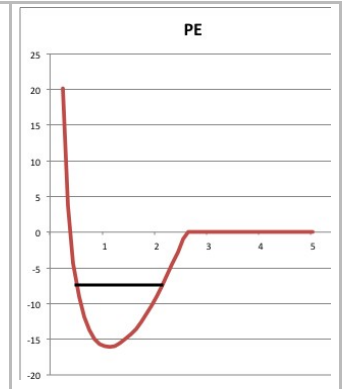
outside the well (to the value of 0 energy).

D. If she wants to climb out of the well and be at 0 kinetic energy at the point  $x = 3$  units, how much energy would she need to gain?

10 units of energy.

E. The skateboarder is actually just an analogy for the cases we are interested in, which are interacting atoms. This is really too simple a model the atoms are impenetrable and will repel if pushed too close together. Instead of the simple well shown above, the atom-atom potential looks more like the one shown in the figure at the right. When the atoms are far apart there is little to no interaction. When they are closer, they are attracted and pulled together. If they get too close they are pushed apart. The potential energy of the interaction looks like the figure at the right.

If the atoms have the energy of -7.5 units as shown by the solid line in the figure, describe their motion and how their potential and kinetic energies change as they move in the well.



The atoms will be oscillating and bound together since their total energy is negative. We can start looking at the oscillation at the point when their separation is 2 units and their KE is 0. They are therefore at rest. Since the well slopes down to the left, there is a force pushing the atoms together. They begin to speed up towards each other, their KE increasing and their PE decreasing until they are approaching each other at their fastest speed at the bottom of the well. At that point their PE is a large negative value and their KE is the positive value needed to bring their total energy up to -7.5 units. They continue approaching each other (since the force at the minimum is 0) but as they get closer the PE rises creating a repulsive force which slows them down. Their PE rises so their KE drops until they reach about  $x = 0.5$  units. There, their PE = their total E (-7.5 units) so their KE equals 0. They start being pushed back apart and the cycle reverses. When they get back to 2, they stop and the cycle begins again.

F. If the atoms have an energy of -7.5 units as shown by the solid line in the figure, would you have to put energy in to separate the atoms or by separating them would you gain energy by separating them? How much? Explain why you think so.

They have a total energy of -7.5 units so to get them apart you would have to put in +7.5 units of energy. This would put them at rest (KE = 0) a large distance apart (PE = 0).