Mathematical Modeling Activity

NEXUS/Physics Workshop
BioQuest 2014, University of Delaware
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See slide 16 of the presentation *Mathematical Modeling Workshop* for background information.

A. It is reasonable to model the shape of the earthworm as a solid cylinder. Using the dimensions of a typical earthworm above, calculate its surface area (ignore the surface areas of the blunt ends in all calculations), volume, and density.

B. If the worm is much longer than it is wide ($L >> R$) is it OK to ignore the end caps of the cylinder in calculating the surface area? How does the surface area and volume of the worm depend on the length of the worm, L, and the radius of the worm, R?

C. For an arbitrary worm of length L, radius R, and density d, write an equation (using the symbols A and or B rather than the numbers) that expresses the number of moles of oxygen the worm absorbs per hour and the number of moles the worm uses per hour. What is the condition that the worm takes in oxygen at a rate fast enough to survive? Does this simple model predict that the typical worm described above absorbs sufficient oxygen to survive?

D.1. Consider the effect of changing the various size parameters of a worm. First consider a worm of length 12 cm that grows by keeping its length the same but increasing its radius. Use a spreadsheet to plot the total oxygen absorbed through the skin of the worm and the total oxygen used by the worm as a function of its length from a radius of 0 cm (not really reasonable) up to a radius of 1 cm. Do the two curves cross? Explain what the crossing means and what its implications are.

D.2. Now consider a worm width 0.64 cm that grows by keeping its width the same but increasing its length. Use a spreadsheet to plot the total oxygen absorbed through the skin of the worm and the total oxygen used by the worm as a function of its length from a length of 0 cm (not really reasonable) up to a length of 50 cm. Do the two curves cross? Explain what the crossing means and what its implications are.

D.3. Write (in symbols) an equation that represents the crossover condition -- that the oxygen taken in per hour exactly equals the oxygen used per hour. Cancel common factors. Discuss how this equation tells you about what you learned about worm growth by doing the two graphs.

E. Our analysis in D was a modeling analysis. An organism like an earthworm might grow in two ways: by just getting longer or isometrically -- by scaling up all its dimensions. What can you say about the growth of an earthworm by these two methods as a result of your analysis in part D? Does a worm have a maximum size? If so, in what sense? If so, find it.

F. In typical analyses of evolution and phylogenetic histories, earthworm-like organisms are the ancestors of much larger organisms than the limit here permits. Discuss what sort of variations in the structure of an earthworm might lead to an organism that solves the problem of growing isometrically larger than the limit provided by this simple model.