Analyzing the many skills involved in solving complex physics problems
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Analyzing the many skills involved in solving complex physics problems

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We have empirically identified over 40 distinct sub-skills that affect a person’s ability to solve complex problems in many different contexts. The identification of so many sub-skills explains why it has been so difficult to teach or assess problem solving as a single skill. The existence of these sub-skills is supported by several studies comparing a wide range of individuals’ strengths and weaknesses in these sub-skills, their “problem solving fingerprint,” while solving different types of problems including a classical mechanics problem, quantum mechanics problems, and a complex trip-planning problem with no physics. We see clear differences in the problem solving fingerprint of physics and engineering majors compared to the elementary education majors that we tested. The implications of these findings for guiding the teaching and assessing of problem solving in physics instruction are discussed. © 2015 American Association of Physics Teachers.  
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I. INTRODUCTION

Problem solving is arguably the most important skill a physicist can have, but in spite of extensive research, progress has been quite limited in finding effective ways to measure and teach this skill.1–3 Our hypothesis is that progress in problem solving has been slowed by the difference between the complexity of the subject and the simplicity of the measurement tools. Usually all that can be measured is whether the person can solve the problem or not. This provides no insight as to what the learner needs to do to improve and no guidance to the teacher as to how to help them. It is like trying to teach a person to play golf when the only information available is whether the ball they hit went into the hole or not. “Just do it better” is an ineffective way to teach either golf or physics problem solving. “Go watch Tiger Woods” (or equivalently, an expert physicist) is only slightly more helpful, because little of what such experts do is evident or readily copied by non-experts.

Although numerous scholars have delineated and classified different elements or sub-skills of problem solving, there was little empirical work to justify that these were correct and complete characterizations and even less work on ways to measure an individual’s level of strength in each sub-skill. Assessing individual sub-skills, rather than rating a person’s overall “problem solving” ability, is necessary for teachers and researchers to understand their students’ current state and to determine effective teaching strategies for improving problem solving.

In this paper, we provide a brief history of problem solving research including work to teach problem solving skills, characterization of the types of sub-skills that are required to solve problems, and assessment of problem solving. This background is followed by a description of the research methodology we used to empirically identify the many different sub-skills used when solving complex problems, the development of the rubric used to determine an individual’s strengths and weaknesses in each sub-skill (their “problem solving fingerprint”), and the different types of evidence collected to support the validity of these skills. This evidence includes expert review of the sub-skills and comparison of a person’s problem solving fingerprint as determined while solving different types of problems including: in class problem solving as observed by their physics instructor, a complex introductory-level mechanics problem; and quantum mechanics problems. Finally, we will discuss implications for teaching including a comparison of physics majors to pre-service elementary teachers followed by the presentation of two individual case studies.

II. BACKGROUND ON PROBLEM SOLVING RESEARCH

The definition of problem solving that will be used in this paper comes from Mayer and Whitrock, who say, “Problem solving is cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver.”4

This definition means the designation of a task as a problem must be based on the solver’s response to the task, rather than the task itself. If a task does not qualify as a problem for a particular individual, then it becomes merely an exercise for them. This is an important distinction. Much of the
classic expert/novice problem solving literature\textsuperscript{5–7} used tasks that were exercises for the experts, producing very different results than studies that used tasks that were authentic problems for both the experts and novices.\textsuperscript{8–10} In tackling authentic problems, experts did not follow an efficient, working-forward approach as described by the classic expert-novice literature. However, experts did spend more time analyzing, planning, and managing their own behavior, demonstrating a more holistic view and systematic problem-solving approach. Their knowledge base also helped them better define the problem and prevented a wide range of oversights that were common to novices.

A. Teaching problem solving

Extensive work has focused on improving students’ problem solving skills in science. Typically problem solving is taught as a whole, utilizing a step-by-step strategy. These attempts have met with, at best, limited success. Some researchers report problem-solving improvement; however, what is meant by improvement in problem solving is not what we’ve defined as problem solving above. For example, some see improvement in demonstration of the prescribed steps, but not in success at finding the correct solutions.\textsuperscript{11,12} Heller and Hollabaugh\textsuperscript{13} do find improvement in correct solutions of problems when students work in cooperative groups, and Heller and Reif\textsuperscript{14} show that students perform better, while worked examples are available as references and they are guided through the solution strategy. However, some researchers found that teaching problem solving strategies, as recommended from the above literature, did not result in improved student performance and that students did not find these strategies beneficial or worth the extra time.\textsuperscript{15,16}

These step-by-step approaches largely come from the same research that defines the quintessential expert who efficiently solves a problem following an eloquent procedure. However, these procedures come from observing experts solving exercises, not problems, so they do not represent what an expert actually does when solving an authentic problem. An additional drawback with students learning to emulate these eloquent solutions is that they are solutions to a typical textbook problem, where there is a clear problem statement, a single correct solution, and all information needed is given. Such exercises are not representative of the problems encountered in real-world situations (known as “ill-structured problems” in the problem-solving literature).

B. Characterizing problem-solving sub-skills

Research in cognitive science, psychology, and math on characterizing specific types of problem solving skills has typically organized them into two to four categories. Mayer and Whitrock\textsuperscript{4} categorize skills into knowledge (“knowing”) and processes (“doing”). Processes include representing, planning/monitoring, executing, and self-regulating. Knowledge includes facts, concepts, strategies, procedures, and beliefs/metacognitive knowledge. Schoenfeld\textsuperscript{6} framed things slightly differently using resources, heuristics, control, and beliefs. Nevertheless, a number of researchers have argued that these are not adequate, and that what it is that people do when they solve problems still needs to be better characterized and understood.\textsuperscript{17–19}

The PISA 2012 problem-solving framework, built on the research discussed above, includes the most complete listing of sub-skills that we have found in the literature;\textsuperscript{3} however, it only focuses on processes, and not knowledge or beliefs and motivations. The PISA Problem Solving Group has identified four sets of processes involved in complex problem solving: exploring and understanding; representing and formulating; planning and executing; and monitoring and reflecting. There are 8–15 separate skills listed under each of these four processes. Their assessment then ranks the value of the four processes by theoretical importance and scores according to the value of the skill. But, there is no indication that this delineation of skills was empirically tested, and their assessment only measures a single overall score.

C. problem solving assessment

The state of assessment of specific problem-solving sub-skills has lagged the characterization. Due to the importance of subject knowledge, the inherent difficulty with distinguishing the many different sub-skills that a person uses to solve a complex problem is made worse when one considers assessment of problem solving sub-skills in a particular subject area like physics. If a student lacks a crucial piece of physics knowledge, it can be impossible to learn about any other sub-skills that they have not yet used while solving the problem. This is the limitation of several problem-solving assessments developed for use in biology, chemistry, and physics.\textsuperscript{20–24} Finally, there are also several assessments designed to evaluate value-added subject-independent competencies in problem solving at the university level. These include: Collegiate Learning Assessment (CLA) performance task and critique-an-argument task;\textsuperscript{25} ACT’s Collegiate Assessment of Academic Proficiency (CAAP) Critical thinking test;\textsuperscript{26} and ETS’s Measure of Academic Proficiency and Progress (MAPP) Reading and critical thinking.\textsuperscript{27} These assessments only provide a single score to reflect a student’s composite ability and there is no evidence that what these assessments measure is relevant to physics problem solving.

III. METHOD

We set out to find an approach that would: empirically identify all the sub-skills that are used in solving a complex problem; measure the strengths of each of those sub-skills in an individual; and test the relevance of those sub-skills to solving physics problems. After substantial exploratory work, we settled on an approach that has a number of novel features.

- We are using a very complex authentic problem, developed by cognitive scientists, to call on a very large number of sub-skills, but it does not involve any science knowledge.
- We determined that it was easier to initially detect a sub-skill by seeing the impact of its absence rather than its presence. When a solver is particularly strong in a specific sub-skill, it can be hard to observe its use, so we analyzed and compared a large number of solvers with an extremely wide range of experiences and sub-skills through cognitive “think aloud” interviews. Once we had identified a specific sub-skill by seeing the difficulties encountered by a solver that lacked that particular sub-skill, we examined indications of the use of that specific sub-skill in the work of the full range of solvers. This allowed us to identify indications of different levels of that sub-skill displayed by
savers, and those indicators were then incorporated into our problem-solving-assessment rubric.

- We developed a scenario-problem-solving-assessment tool, where the solver is put in a scripted scenario as the evaluator of two characters who are working through the complex problem. This allows the evaluation to continue, even after the solver has revealed critical weaknesses that would terminate the solution progress if they were trying to solve the problem on their own.

The entire methodology was rather lengthy and extends well beyond physics, and so will be fully presented in another publication. Here we will present an abbreviated summary and the results obtained that are relevant to physics problem solving.

A. Identifying sub-skills of problem solving

We used the problem Rescue at Boone’s Meadow from the Jasper Woodbury Series, developed by the Cognition and Technology Group at Vanderbilt University. This problem is an everyday complex problem with an embedded data design—all content knowledge is provided. The story involves transport of an injured eagle from a remote location using a variety of constrained transportation options. Due to the carefully designed nature of the problem, this assessment is a true problem for every solver that has been evaluated, from high-school dropouts to physics professors.

A script was created that features two interns talking as they work through the solution to the problem. A brief introduction is provided before the eagle rescue story, explaining to the solver that they are to play the role of a manager in a company who is responsible for choosing which one of two good summer interns, Jasmine or Sara, would be offered a permanent position with the company. The permanent position would require strong problem solving skills. To assist in the manager’s decision, the interns would be asked to solve a very involved problem together.

As they work through the script, the solver is presented with 40 open-ended questions that are placed at crucial points throughout the interns’ discussion. These questions ask the solver’s opinion on how the conversation and problem-solving process between the interns is progressing. The questions also ask the solver for items of factual knowledge, planning, procedures, calculations, and reflections. The intern script and the questions evolved over time as they were used in interviews. They started relatively open, but as particular sub-skills were identified, the script and the questions were modified to more specifically target the information needed to identify the solver’s level of competence in each sub-skill.

A set of 18 follow-up questions is placed at the end of the intern script. These questions give the solver a chance to discuss other ideas they may have had that the script did not bring out, probe their ideas about alternative solutions, evaluate the solver’s metacognitive sub-skills and ask the solver to analyze, in detail, each intern’s problem solving skills.

I. The Sub-skills

Through interviews with the scripted eagle-rescue problem, we found 44 sub-skills that are needed to describe fully the problem-solving process used by subjects (Table I). For the purposes of this research, the definition of a sub-skill is: anything that can affect the subject’s ability to solve the problem. These sub-skills emerged during the first 20 or so interviews and no additional sub-skills emerged during 11

<table>
<thead>
<tr>
<th>Knowledge—have</th>
<th>Beliefs, expectations, and motivation</th>
<th>Processes—do</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Math—basic add/sub/mult/div</td>
<td>• Confidence</td>
<td>• Acquires Info 1st time through</td>
</tr>
<tr>
<td>• Math—equation formation</td>
<td>• Attribution (takes responsibility for their actions)</td>
<td>• Plan ideas (What—ask questions)</td>
</tr>
<tr>
<td>• Reading comprehension</td>
<td>• Judgment of information based on the source</td>
<td>• Plan way to get answer (How)</td>
</tr>
<tr>
<td>• Spatial—2-D mapping</td>
<td>• Wants to solve the problem for self</td>
<td>• Plan—big picture (Visualization)</td>
</tr>
<tr>
<td>• Previously known facts</td>
<td>• Wants to solve the problem for interviewer</td>
<td>• Keep problem framework in mind</td>
</tr>
<tr>
<td>• Real World knowledge</td>
<td>• Wants to succeed on the “test”</td>
<td>• Connect steps and pieces</td>
</tr>
<tr>
<td>• Knowledge of own Strengths</td>
<td>• Interested in the context of the problem</td>
<td>• Check calculations of others</td>
</tr>
<tr>
<td>• Knowledge of own Weaknesses</td>
<td>• Enjoyed solving the problem</td>
<td>• Aware of how others helped</td>
</tr>
<tr>
<td>• Number Sense</td>
<td>• Enjoyed analyzing interns</td>
<td>• Meta-process – step outside of problem solving to see if own actions are useful.</td>
</tr>
<tr>
<td>• Estimation</td>
<td>• Enjoyed complete experience</td>
<td>• Skepticism</td>
</tr>
<tr>
<td>• Ability to analyze others (interns)</td>
<td>• Real life vs. student</td>
<td>• Estimation</td>
</tr>
<tr>
<td></td>
<td>• Careful/Thorough</td>
<td>• Creativity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Adaptability (shifts direction easily)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Can throw out useless info</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Judgment of reasonable issues</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Judgment of importance of number values (is it material)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Tie in personal experiences</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Tie in info provided by another</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Scientific Process (each step justified with evidence not by gut feeling)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Remember previously noted facts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Remember what s/he has calculated or reasoned.</td>
</tr>
</tbody>
</table>

Note: For a description of the specific behavior associated with each category see the problem solving rubric (Ref. 29).
IV. VALIDATION STUDIES

We carried out validation studies to establish:

(1) That these sub-skills have face validity, in that faculty who are both researchers and teachers see the list of sub-skills as important to solving problems in their disciplines and are skills they see as valuable for students in their courses to have.

(2) That people use a similar set of sub-skills with the same level of competence when solving physics problems as what we measure with the scripted eagle rescue and associated rubric tool, henceforth referred to as the Assessment of Problem-Solving Skills (APSS).

A. Expert review

Early in the project, interviews were conducted with eleven people who are considered highly successful in their respective fields. These included physicists, biophysicists, biochemists, biologists, chemists, mathematicians, accountants, and business owners. These interviews consisted of asking the interviewee to summarize how they tackle difficult problems in their field, including different types of problems. The eleven interviews revealed that the skills that were indicated as necessary were quite consistent across disciplines (e.g., connect steps and pieces). The only obvious differences were a limited set of skills associated with specific scientific knowledge or research processes necessary in that particular field. This point came out particularly clearly when talking to biochemists or biophysicists, people who could provide a perspective on solving problems across different disciplines.

Later in the project, after identifying the sub-skills listed in Table I, we showed them to 19 university and high-school teachers in a range of science disciplines. These teachers were asked how well the list captured the problem-solving sub-skills that they felt were important. These teachers all agreed that all of these sub-skills, except enjoyed analyzing internships, are skills that they value in their discipline and consider important for solving problems. They did not identify any additional sub-skills beyond those on our list.

In an informal test, we also used the APSS to measure the sub-skills of 24 student volunteers (10 were interviewed while completing the APSS and 14 provided written responses) from three relatively small physics courses where there was extensive instructor-student interaction, and then asked the instructors for assessments of each student’s strengths and weaknesses. The instructors were able to provide detailed assessments for every student, and these were all consistent with what we had measured. However, the APSS, used with either administration method (1–1.5 h interview or written response), provided more detail, and all of the instructors indicated that those results explained behaviors and student difficulties they had seen during the semester but had not understood.
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Table II. Sub-skills that could not be scored with both assessment methods.

<table>
<thead>
<tr>
<th>Scored with APSS, Not</th>
<th>Scored in Pyramid interview, Not with APSS</th>
<th>Scored in APSS interview, Not Quantum interview</th>
<th>Scored in Quantum interview, Not APSS interview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previously known facts</td>
<td>Math knowledge-geometry</td>
<td>Previously known facts</td>
<td>We did not attempt to score the many Knowledge components involved in solving quantum problems over the course of a semester.</td>
</tr>
<tr>
<td>Ability to analyze interns</td>
<td>What think should know inhibits effective solving</td>
<td>Ability to analyze interns</td>
<td></td>
</tr>
<tr>
<td>Enjoyed Analyzing Interns</td>
<td>Rounding</td>
<td>Enjoyed Analyzing Interns</td>
<td></td>
</tr>
<tr>
<td>Adaptability</td>
<td>Spatial—3-D visualization</td>
<td>Estimation</td>
<td></td>
</tr>
<tr>
<td>Acquires information the first time through</td>
<td></td>
<td>Interested in the context of the problem</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Number Sense</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Spatial—mapping</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Careful/Thorough</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Judgment of relevance of number values</td>
<td></td>
</tr>
</tbody>
</table>

B. Comparison with solving a mechanics problem

We compared the APSS results with the sub-skills used by students solving a mechanics problem during think-aloud interviews. We chose the Pyramid of Giza problem\(^{31}\) which requires the use of Newton’s Laws as well as conservation of energy. This problem takes about two hours to solve and is designed to require a larger number of sub-skills than typical textbook problems.

Five student volunteers were recruited from second term algebra- and calculus-based introductory physics courses. These students and the interviewer (WA) had not previously met. The students were each given the APSS and wrote out answers to the questions and brought them to their pyramid-solving interview. An identifier was added to each completed response so that it was anonymous to the grader. The students then participated in think-aloud interviews where they worked through the Pyramid of Giza problem and were scored on the problem solving sub-skills they used.

During the interviews, the interviewer offered to provide any physics content that they requested, but no solution ideas. In all cases, after an hour of working, students were exhausted and their productivity was visibly reduced. For this reason, the interviews were split into one-hour segments conducted a week apart. All of the students progressed to a point where they considered the problem complete, at which point they were asked to reread the question to make sure they had answered everything. They then realized that they had not and began working on the next task. At the end of the last interview, students were asked follow-up questions about their solving process.

The methodology for scoring student’s problem-solving sub-skills was the same as with the APSS interviews; the interviewer noted the strength of problem solving sub-skills as they surfaced during the interviews. Two months after the last of the pyramid interviews, the APSS written responses were graded (by WA). Because of the somewhat limited information available with the written responses, between one and seven of the sub-skills could not be scored for each student, and the resolution of the scoring was reduced. In response, scoring was condensed to a three-point scale.

As shown in the first two columns of Table II, there were five sub-skills that did not come out in the pyramid problem that students did use with the APSS, and there were four sub-skills that were needed for the pyramid problem that are not used with the APSS.

While the pyramid problem uses more sub-skills than normal textbook problems, it does not require several of the problem solving sub-skills that are probed with the APSS, such as acquires info the first time through, previously known facts, and Adaptability. A notable sub-skill that only showed up on the pyramid problem was what think should know inhibits effective solving. This sub-skill refers to the difficulty students had when they thought they should know a formula or method, whether it existed or not.

After scoring the anonymous APSS responses, it appeared obvious which belonged to which student, and this identification was confirmed when the written responses were unblinded. Each student had a pattern of strengths and weaknesses in solving the pyramid problem, and that same pattern, their “problem-solving fingerprint,” was readily apparent on the APSS; Fig. 1 shows the comparison for a typical student. Of the 25 sub-skills that could be scored with both assessments, 17 had identical scores, and only two had scores that differed by more than 1. The other students were similar. Table III shows the probability of the level of agreement seen for all students to be $\gg 0.001$. (See Ref. 32 for more detail.)
C. Comparison with solving quantum mechanics problems

The most rigorous validation study was a comparison of students’ APSS results with their sub-skills measured while solving quantum mechanics (QM) problems, where the scoring of the APSS and QM assessments were carried out by different people. An independent researcher, S. McKagan, for the purposes of another study, had interviewed six QM students doing problems every two weeks over the course of a semester. After three days of training with the 5-point scoring rubric for evaluating sub-skills, McKagan used the rubric to score the students sub-skills in solving QM problems by reviewing the videos of her previous QM interviews. It was possible to score the students in 31 of the 40 sub-skills contained in the scoring rubric at that time (Table II).

Five of these six students volunteered to participate in APSS interviews with WA in which their sub-skills were measured while solving the eagle-rescue problem. The results from the two assessments of these students’ problem solving sub-skills were then compared. The two assessments agreed within ±1 point out of 5, on 86% of the 138 items scored by both (5 students x 28 sub-skills; not all 31 sub-skills could be scored for every student, with typically 3 missing for each.) For 30% of the sub-skills, the two scores were identical (Table III).

Each person’s skill set was unique and consistent between the two very different contexts, as illustrated in Figs. 1 and 2. This similarity can also be seen in the two interviewer’s characterizations of individual students, as shown in the following example.

Quote from QM interview:
“she seemed to view learning [as] how to accept every weird thing we told her… she thought the first step was to accept things and the second step was to try to understand them. She always rethought her ideas when another student suggested something, although she maintained enough skepticism to recognize that other students were often wrong.”

Quote from eagle rescue interview:
“Always had a knee jerk response which was not always good but then, on her own, she considers carefully and comes up with the right answer….she’ll consider whatever is thrown out there, decisions are based on the most logical answer. If a suggestion does not make sense after careful consideration, she holds onto her beliefs. Very reliable”

This is just an example from one student; however, the summaries from each student were similarly as consistent, even when there was as much as a year difference between when the student participated in the quantum mechanics interviews and the APSS interview.

These studies showed that all but three of the sub-skills that we identified with the APSS were used in solving at least one of these two physics problems. Two of the three missing sub-skills, previously known facts and ability to analyze interns (where one replaces “interns” with “collaborators” or “employees”) would be needed for many authentic real-world problems a physicist might encounter. These results also demonstrate that a person’s level of competency in these

<table>
<thead>
<tr>
<th>Number of sub-skill scores for which the two scores were within Δ (out of 92).</th>
<th>Δ = APSS - Pyramid scores</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>49</td>
<td>0</td>
<td>≤0.0001</td>
</tr>
<tr>
<td>85</td>
<td>±1</td>
<td>≤0.0001</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of sub-skill scores for which the two scores were within Δ (out of 138).</th>
<th>Δ = APSS - QM scores</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>0</td>
<td>0.0019</td>
</tr>
<tr>
<td>119</td>
<td>±1</td>
<td>≤0.0001</td>
</tr>
<tr>
<td>131</td>
<td>±2</td>
<td>≤0.0001</td>
</tr>
</tbody>
</table>

Note: p-values calculated by finding the chance that scores will randomly agree within Δ and using the binomial distribution to find the standard deviation.
sub-skills transfers between different contexts and problem types.

V. IMPLICATIONS FOR TEACHING

We hypothesize that problem solving is readily teachable, but not as a single skill. There are many specific sub-skills that must be addressed, one at a time, similar to teaching physics content knowledge. The work presented here can help an instructor recognize and focus activities on individual sub-skills to help students address weaknesses and to build up a complete set of problem solving sub-skills. In addition, this work will help instructors recognize where different students need particular types of help.

A. Physics/engineering students vs pre-service elementary teachers

Examples of differences between students were clear when we compared how physics and engineering majors did relative to elementary education majors on the APSS (Table IV).

Physics and engineering majors were able to find solutions to the problem typically using strong math, planning, and judgment sub-skills; however, these students did not demonstrate strong knowledge of strengths/weaknesses sub-skills, or engage in meta-processing (stopping and thinking about their current actions). We believe this is because their personal skill set, with which they are proficient, has been adequate for learning in their major thus far. When faced with new challenges, missing sub-skill(s) become a problem and metacognitive sub-skills become important to both identify their weaknesses and facilitate the process of correcting weaknesses. Physics majors who were less successful in their courses tended to be weaker in these sub-skills than the most successful physics students.

Many elementary-education majors have almost an opposite set of strengths and weaknesses from the physics majors. The elementary-education majors who were interviewed had many more weaknesses. The particular weaknesses varied but none had a strong, broad skill set resembling a physics major. Surprisingly, almost all of these elementary-education students had strong metacognitive sub-skills; some were very careful and thorough when working on a problem and a few even engaged in meta-processing. These sub-skills helped them compensate for their many other weaknesses in the knowledge and processes categories.

It is often claimed that elementary-education majors struggle with science and/or math because they have low confidence in their abilities to do science or math. Out of the eight students who were interviewed only two had low confidence in their math/science abilities, although all were, in fact, weak in math/science (not particularly surprising when considering their lack of experience in math and science).

B. Sub-skills taught in the classroom

These results illustrate a more general educational challenge. Many of the sub-skills listed in Table I are not taught, practiced, or assessed in regular physics courses, and, not surprisingly, many of those same sub-skills are ones in which all the students we measured tend to be weak. This holds true for both physics majors and elementary-education majors, although the courses they take, and the respective sub-skills they practice in those courses, are quite different.

When thinking about how to teach problem solving, it is useful to consider that not all sub-skills are required to solve all problems. Back of the chapter textbook problems are short and usually well defined, so they require only a handful of the sub-skills that have been identified during this study as being needed to solve authentic problems. Many of the sub-skills we have listed go unpracticed and unassessed in typical physics classes but are important in the workplace or the laboratory.

With the awareness that there are different, specific-component problem-solving sub-skills and that certain combinations are effective for different types of problems, instructors can focus on specific sub-skills by giving students appropriate problems that will require the use of those skill(s), and provide assessment and feedback targeted to those sub-skills.

C. Individual student examples

As an example, a physics major at UNC in introductory physics (we’ll call him Ben) demonstrated very strong math and number sense sub-skills and considered a range of details that may affect the solution while solving the eagle-rescue problem during an interview. He remembered all the information that he’d acquired and numbers he’d calculated, and kept the problem framework in mind. Ben was also very verbally critical of the interns. However, he had very weak reading comprehension, was not actually skeptical of the information offered by the interns (he took what they said for granted and never checked), and was unable to judge reasonable issues for the problem. Ben was aware that he was weak in some areas of problem solving so added extra cushion to his answers just in case he had made a mistake. The professor’s analysis identified Ben as a weak problem solver but a very conscientious student with strong math sub-skills. Over time the professor had discovered that Ben appeared skeptical in group environments, yet never thought through what was offered by others, and was often being led astray. The one feature the professor had not identified was Ben’s weak reading ability. The professor commented that this

<table>
<thead>
<tr>
<th>Common strengths</th>
<th>Common weakness</th>
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<tbody>
<tr>
<td>Physics/engineering students</td>
<td>Math—equation formation, Math—number sense, planning, creativity, and judgment</td>
</tr>
<tr>
<td>Pre-service elementary teachers</td>
<td>Knowledge of strengths/weaknesses, and Careful and thorough</td>
</tr>
</tbody>
</table>
knowledge could have helped him do a better job of addressing Ben’s problems in class. Most of what the professor surmised about Ben took the better part of a semester of careful observation to identify. If he’d known at the beginning of the semester where Ben was weak, he could have addressed these issues early on.

Another example of the potential value to instructors of recognizing this detailed set of problem-solving sub-skills is provided by a student we will call Janet. In class, Janet did quite well in group activities, especially those involving calculations and/or working through data. She was a much sought-after group member. When interacting with her group, it was clear that Janet always knew how they’d arrived at their solution while the other students often had missed some aspect. However, when working on an individual project involving the synthesis of ideas from two sources, Janet struggled. This seemed quite surprising and hard to understand considering her overall class performance. At the end of the semester, Janet completed the eagle-rescue assessment interview and this made it clear that Janet was extraordinarily weak at formulating questions on her own. After 30 min of making no progress on the first question in the script, which required her to indicate what the interns needed to figure out, the interviewer encouraged her to move on to see what the interns would do. She then went through the script answering the questions easily. If the interns had any ideas about what might be useful, Janet was able to figure out how to find that information and carry out whatever calculation was needed. Throughout the interview, however, she demonstrated an inability to ask her own questions—what might be needed for the next step in the solution—but had no difficulty in finding the answers to any question posed and evaluating its usefulness. Janet graduated with straight A’s in spite of being unable to make progress on any problem that did not have a clearly articulated question. This fairly serious weakness was never addressed, because she was in a major that is taught almost 100% in group format, so she was able to rely on group members to formulate questions, and normal course exams involve well-articulated questions.

D. Discussion

To learn any skill a person must practice it. Here, we have described which sub-skills are commonly practiced by students doing textbook problems and identified those sub-skills that are rarely needed to be successful in the typical undergraduate physics classroom. Being aware of this allows a physics instructor (teaching any level of physics) the opportunity to consider where s/he may or may not incorporate practice of these other sub-skills. Whether one considers these sub-skills as program-level or course-level goals, it is important to identify where students will learn these sub-skills if they are part of the desired outcome.

We often have colleagues ask us if there are specific skills that are more general or more important than others. Different problems require different sub-skills to solve them. Similarly, different sub-skills have varying levels of importance for successful solution of different problems. As noted above we did find that different sets of sub-skills defined a strong physics major compared to a strong pre-service elementary teacher. The relative importance of sub-skills is defined by the particular problem, similar to the definition of a problem being defined by the solver and not just the task.

It would be interesting to compare skills and processes in the problem-solving literature to our sub-skills and categories. Here, we offer a couple of examples. In most cases, the skills described in the literature subsume many sub-skills. One example is “heuristics,” which would fit within our category of knowledge that students have. Heuristics are “rules of thumb” or strategies such as exploiting analogies or working backward and are something students learn to use to solve certain categories of problems making them domain specific. As mentioned earlier, the knowledge category is basically infinite since it contains content knowledge. A second example would be the skills identified in Docktor and Heller’s rubric.21 Three of these skills, “physics approach, specific application of physics, and mathematical procedures,” are knowledge, measures of specific content sub-skills needed to solve each problem while “logical progression” would closely map to scientific process (each step justified with evidence not by gut feeling).

VI. CONCLUSION

We have empirically identified 44 distinct sub-skills that affect a person’s ability to solve problems in many different contexts. We see that a person brings the same problem-solving fingerprint representing their strengths and weaknesses across these sub-skills to problems they encounter in different contexts. In particular, we see students displaying nearly the same sub-skill fingerprint when solving both a mechanics problem and introductory quantum mechanics problems as they displayed when solving a non-physics problem. The existence of so many distinct important sub-skills explains why it has been so difficult to teach or assess problem solving as a single distinct skill, and this work will allow physics teachers to better target their problem-solving instruction. A goal of future work is to create assessment tools that will not require as much time and training to use and will provide reliable measures of at least many, if not all, of these sub-skills used in physics problem solving.

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